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APPLICATIONS OF NEW TRANSFORM "IMAN TRANSFORM" TO MECHANICS, ELECTRICAL CIRCUITS AND BEAMS PROBLEMS

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Abstract

In this work, we apply a new integral transform to solve linear ordinary differential equations namely Iman transform, in Particular we apply Iman transform technique to solve mechanical electrical circuits and beam problems.

Keywords: Iman Transform – Differential Equations – Application.

INTRODUCTION

Many problems of physical interest are described by ordinary or partial differential equations with appropriate initial or boundary conditions.

These problems are usually formulated as initial value problems, boundary value problems, or initial – boundary value problems that seem to be mathematically more rigorous and physically realistic in applied and engineering sciences.

Iman transform method is particularly useful for finding solutions of these problems. The method is very effective for the solution of the response of a linear system governed by an ordinary differential equation to the initial data.

This paper deals with the solution of ordinary differential equations and system of ordinary differential equations that arise in mathematical physical and engineering sciences. The applications of Iman transforms to the initial – boundary value problems are also discussed in this paper.

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A new transform called Iman transform [1] defined for functions of exponential order is considered. We consider functions in the set *A* defined by:

$$A=\{f(t): \exists M,\ k1,\ k2>0,\ |f(t)|<Me^{\frac{|t|}{k_j}};\ \ t\in(-1)^j \times [0,\infty[\ \} \qquad (1)$$

For a given function in the set *A* , the constant M must be finite number and $k1$, $k2$can be finite or infinite. Iman transform denoted by the operator$I(.)$ is defined by the integral equation

$$\text{I}[f(t)] = L(v) = \frac{1}{v^2}\int_0^\infty f(t)e^{-tv^2}dt, \quad k1\le v\le k2,\ t\ge 0 \qquad (2)$$

The variable *u* in this transform is used to factor, the variable *t* in the argument of the function *f* . this transform has deeper connection with the Laplace transform. We also present many differences in the properties of this new transform and Sumudu transform, except a few properties.

The purpose of this study is to apply this interesting new transform and its efficiency into mechanics, electrical circuits and beam problems.

Theorem:

Let *L* (*v*) be Iman transform of *f* (*t*), *I*(*f* (*t*)) = *L*(*v*), then

(i) $\text{I}\left[\frac{\partial f(x,t)}{\partial t}\right] = v^2 L(x,v) - \frac{1}{v^2} f(x,0)$

(ii) $\text{I}\left[\frac{\partial^2 f(x,t)}{\partial t^2}\right] = v^4\, L(x,v) - f(x,0) - \frac{1}{v^2}\,\frac{\partial f(x,0)}{\partial t}$

Proof:

(i) By the definition we have:

Integrating by parts, we get:

$$\text{I}[f'(t)] = \frac{1}{v^2}\int_0^\infty f'(t)e^{-tv^2}dt$$

$$\text{I}\left[\frac{\partial f(x,t)}{\partial t}\right] = v^2 L(x,v) - \frac{1}{v^2} f(x,0)$$

(ii)

$let\ g(x,t) = f'(x,t)\ \ then$

$$\text{I}\left[\frac{\partial g(x,t)}{\partial t}\right] = v^2\text{I}[g(x,t)] - \frac{1}{v^2} g(x,0) \quad \text{using (i) we find that:}$$

$$\text{I}\left[\frac{\partial^2 f(x,t)}{\partial t^2}\right] = v^4\, L(x,v) - f(x,0) - \frac{1}{v^2}\,\frac{\partial f(x,0)}{\partial t}$$

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1. Applications of Iman Transform

Application to Mechanics

A particle P of mass 2 grams moves on the *X* axis and is attracted towards origin *O* with a force numerically equal to 8*X*. If it is initially at rest at *X* =10 find the position at any subsequent time, assuming,

(a) No other forces act.

(b) A damping force numerically equal to 8 times the instantaneous velocity acts.

(a) Choose the positive direction to the right. when $X > 0$, the net force is to the left and must be given by -8*X* when $X < 0$ the net force is to right and must be given by -8*X. Hence* in either case the net force is -8*X*

Then by Newton's law. (Mass). (Acceleration)=Net force

$$2\frac{d^2 X}{dt^2} = -8X \Longrightarrow \frac{d^2 X}{dt^2} + 4X = 0 \qquad (3)$$

The initial conditions are:

$$X(0) = 10 \, , X'(0) = 0 \qquad (4)$$

Taking Iman transform of (3) and using conditions (4), we have,

$$if \quad X' = I(X) = v^4 X'(v) - 10 + 4X'(v) = 0$$

$$\Longrightarrow X'(v) = \frac{10}{v^4 + 4} = 10\left(\frac{1}{v^4 + 4}\right)$$

Then

$$X(t) = 10\, I^{-1}\left(\frac{1}{v^4 + 4}\right) = 10\, cos2t$$

The amplitude [maximum displacement from 0] is 10. the period [time for a complete cycle] is π. The frequency [number of cycles per second] is $\frac{1}{\pi}$

2. Applications to Electrical Circuits

(a) An inductor of 2 henrys, a resistor of 16 ohms and a capacitor of 0.02Farads are connected in series with an e.m.f of E volts at $t = 0$. The charge on the capacitor and current in the circuit are zero. Find the charge and Current at any time $t > 0$ if E= 300 (volts).

Solution

Let *Q* and *R* be the instantaneous charge and current respectively at time *t.* By Kirchboff"s laws, we have

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$$2\,\frac{dR}{dT} + 16R + \frac{Q}{0.02} = E \qquad (5)$$

Or Since R= $\frac{dQ}{dt}$

$$2\,\frac{d^2\,Q}{dt^2} + 16\frac{dQ}{dt} + 50\,Q = E \qquad (6)$$

With the initial conditions

$$Q(0) = 0\,, \qquad R(0) = Q'(0) = 0$$

$$if\ E = 300\ then\ (6)\ \ becomes$$

$$\frac{d^2\,Q}{dt^2} + 8\frac{dQ}{dt} + 25\,Q = 150$$

Then taking Iman transform, we find

$$v^4\,Q'(v) - Q(0) - \frac{1}{v^2}\,Q'(0) + 8\left[v^2 Q'(v) - \frac{1}{v^2}Q(0)\ \right] + 25\,Q'(v) = \frac{150}{v^4}$$

Then

$$Q(t) = I^{-1}\,[Q'(v)] = 6 - 6e^{-4t}cos3t - 8e^{-4t}\sin 3t$$

And

R = $\frac{dQ}{dt} = 50e^{-4t}\sin 3t$

Application to Beams:

Abeam which is hinged at its ends, *x* = 0, and *x* = *N* carries a uniform loud w_0 per unit length. Find the deflection at any point.

Solution

The differential equation and boundary conditions are,

$$\frac{d^4\,y}{dx^4} = \frac{w_0}{ER} \qquad ,0 < x < N \qquad (7)$$

$$y(0) = 0\,, y''(0) = 0, y(N) = 0, y''(N) = 0 \qquad (8)$$

Where E is young's modulus, R is the moment of inertia of the cross section about an axis normal to the plane of bending and ER is called the flexural rigidity of the beam.

Some physical quantities associated with the problem are $y'(x), M(x) = ER\ y''(x)\ and\ \ S(x) = M'(x) = \ ER\ y'''(x)$ which respectively represent the slope, bending moment, and shear at a point.

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Taking Imani transforms of both sides of (7), we have,

$$v^8\, L(v) - v^4 y(0) - v^2 y'(0) - y''(0) - \frac{1}{v^2}\, y'''(0) = \frac{w_0}{v^4\, ER} \qquad (9)$$

$$y'(0) = c_1\,, y'''(0) = c_2$$

Using the first two conditions in (8) and the unknown conditions

$$L(v) = \frac{c_1}{v^6} + \frac{c_2}{v^{10}} + \frac{w_0}{v^{12}\, ER}$$

Inverting to find

$$y(x) = c_1 x + \frac{c_2 x^3}{6} + \frac{w_0 x^4}{24\, ER}$$

From the last two conditions in (8), we find:

$$c_1 = \frac{w_0 N^3}{24\, ER}$$

$$c_2 = -\frac{w_0 N}{2\, ER}$$

Thus the required deflection is,

$$y(x) = \frac{w_0}{24\, ER} x\, (N - x)(N^2 - Nx - x^2)$$

It is now possible to calculate the bending moment and shear at any point of the beam, and in particular, at the ends.

CONCLUSION

Application of Iman transform to mechanics, electrical circuits and beam problems has been demonstrated.

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Table of Functions and their Iman Transform

$\boldsymbol{f(t)}$	$\boldsymbol{I[f(t)] = F(p)}$
1	$\boldsymbol{\frac{1}{p^4}}$
t	$\boldsymbol{\frac{1}{p^6}}$
t^2	$\frac{2!}{\boldsymbol{p^8}}$
$n \in N t^n$	$\frac{n!}{\boldsymbol{p^{2n+4}}}$
e^{at}	$\boldsymbol{\frac{1}{p^2(p^2-a)}}$
$\sin(at)$	$\boldsymbol{\frac{a}{p^2(p^4+a^2)}}$
$\cos(at)$	$\boldsymbol{\frac{1}{p^4+a^2}}$