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# VARIATION OF EQUITABLE COLOR CLASS DOMINATION NUMBER AFTER THE LINK REMOVAL IN GRAPHS

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**Abstract**

Let $G = (V, E)$ be a connected graph. Assume a group $CC$ containing colors. Let $\tau : V(G) \to CC$ be an equitably colorable function. A dominating subset $S$ of $V$ is called an equitable color class dominating set if the number of dominating nodes in each color class is equal. The least possible cardinality of an equitable color class dominating set of $G$ is called the equitable color class domination number itself. It is indicated by $\gamma_{ECC}(G)$. In this paper, we study the changing and unchanging of $ECC$ Domination number after the link removal.

**Keywords:** Dominating Set, Equitable Coloring, Color Class ($CC$), Equitable Color Class ($ECC$), Equitable Color Class Dominating Set, Link Removal

**2020 Mathematics Subject Classification:** 05C15, 05C69

## 1. INTRODUCTION

The study of the effect of removing a link on any graph theoretic parameter has interesting applications in the network context. That is, analyzing the removal of a link is more vital as an important consideration in the topological design of a network is fault tolerance. The behavior of a network in the presence of a fault can be analyzed by determining the effect that removing a link (link failure) from its underlying graph $G$ has on the fault-tolerance criterion. A detailed study of changing and unchanging domination is given in Chapter 5

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of Haynes et. al [6]. Further, The semi-expository paper by Carrington et. al. [2] surveyed the problems of characterizing the graphs G into three classes based on link removal.

If a link is deleted, the value of $\gamma$ may increase or decrease or remain unaltered. Therefore the link set $E$ can be partitioned into the subsets $E^-$, $E^{0,}$ and $E^+$ where

$$E^0 = \{uv \in E : \gamma(G - uv) = \gamma(G)\}$$
$$E^+ = \{uv \in E : \gamma(G - uv) > \gamma(G)\}$$
$$E^- = \{uv \in E : \gamma(G - uv) < \gamma(G)\}$$

Several results on links belonging to the above subsets are given in [6]. In this chapter, we initiate a similar study corresponding to the equitable color class domination number of a graph.

## 2. DEFINITIONS AND NOTATIONS

**Definition 2.1:** [5] In a graph $G = (V, E)$, a subset $S$ of nodes is a dominating set if every node in $V - S$ is adjacent to some node in $S$. The least possible cardinality of the dominating set of $G$ is called its domination number and it is indicated by $\gamma(G)$.

**Definition 2.2:** [8] In a graph $G$, adjacent nodes don't ordain the same color is known as proper coloring. The least possible number of colors used to color a graph $G$ is known as its chromatic number and it is indicated by ▯$(G)$.

**Definition 2.3:** A subset of nodes ordained to the same color is known as a color class.

**Definition 2.4:** [8] In a graph, adjacent nodes don't have the same color, and the difference between the cardinality of color classes is $\leq 1$ is called an equitable coloring graph. The least possible number of colors used to equitably color a graph $G$ is known as its equitable chromatic number and it's indicated by ▯$_E(G)$.

**Notation 2.5:** Let $\mathcal{X}$ be any real number. Then $\lfloor\mathcal{X}\rfloor$ indicates the greatest integer $\leq \mathcal{X}$ and $\lceil\mathcal{X}\rceil$ indicates the smallest integer $\geq \mathcal{X}$.

**Notation 2.6:** If $a$,$b$ be the integers and $n > 0$ then $a \equiv b$ (mod $n$) indicates $n|a$-$b$.

## 3. PRIMARY RESULTS

**Definition 3.1:** [4] Let $G$ = ($V$, $E$) be a connected graph. Assume a group $CC$ containing colors. Let $\tau$: $V(G) \rightarrow CC$ be an equitably colorable function. A dominating subset $S$ of $V$ is called an equitable color class dominating set if the number of dominating nodes in each color class is equal. The least possible cardinality of an equitable color class dominating set of $G$ is the equitable color class domination number itself. It is indicated by $\gamma_{ECC}$ $(G)$.

**Theorem 3.2:** [3] If $G$ = ($V$, $E$) be a connected graph then $\gamma_{ECC}(G)$ = $k$▯$_E$ where $k \in \mathbb{N}$ and ▯$_E$ be the equitable chromatic number of $G$.

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**Proof:** let $G$ = ($V$, $E$) be a connected graph and $v_1, v_2, v_3, \ldots, v_n$ be the nodes of $G$. The equitable chromatic number of $G$ is ⍰$_E$ Let $\tau$: $V(G) \to CC$ be an equitably colorable function where $CC = \{1,2,3,4, \ldots\ldots,$ ⍰$_E\}$. Choose dominating nodes like a pair of ⍰$_E$ number of different color nodes. So, the number of dominating nodes in each color class is equal. Let $k$ be the minimum number of pairs to dominate a graph $G$. Hence, the equitable color class domination number of a graph $G$ is $k$⍰$_E$ where ∈ ℕ.

**Example 3.3:** A simple example of finding $\gamma_{ECC}$ of a general graph $G$.

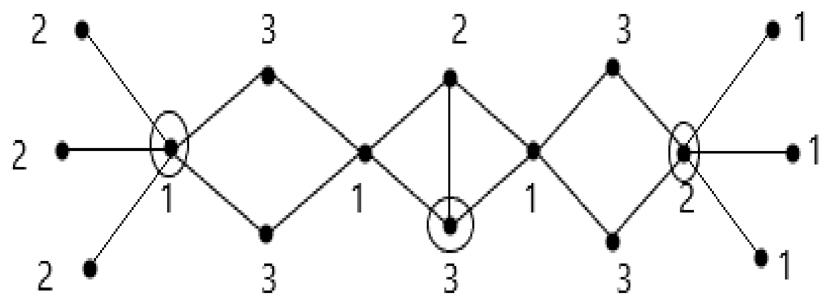


**Fig.3.1**

⍰$_E$ of the general graph $G$ is 3 and chosen dominating nodes in each color class are equal and one, $\gamma_{ECC}$ $(G)$ = 3.

**4 LINK REMOVAL:**

We observe that the $ECC$ domination number $\gamma_{ECC}(G)$ of a graph $G$ may increase or decrease or remain unchanged when a link is removed from $G$. Depending the $ECC$ domination number of the graph $G$ after removing the link, the link set $E(G)$ is partitioned into three subsets $E^-_{ECC}$, $E^0_{ECC}$ and $E^+_{ECC}$ as follows.

$$E^0_{ECC} = \{uv \in E : \gamma_{ECC}(G - uv) = \gamma_{ECC}(G)\}$$
$$E^+_{ECC} = \{uv \in E : \gamma_{ECC}(G - uv) > \gamma_{ECC}(G)\}$$
$$E^-_{ECC} = \{uv \in E : \gamma_{ECC}(G - uv) < \gamma_{ECC}(G)\}$$

In this section, we investigate the properties of the above sets.

**Example 4.1:**

$(i)$ Let $K_n$ be the complete graph with $n$ nodes and $\frac{n(n-1)}{2}$ links. $\gamma_{ECC}(K_n) = n$. After the removal of the link, the number of links and $\chi_E$ is decreased by 1. Suppose the link $uv$ is removed then the nodes $u$ and $v$ have ordained the same color. Hence, $\gamma_{ECC}(K_n - uv) = n - 1$. The value of $\gamma_{ECC}$ is decreasing by 1 for each link removal. Hence, $E(K_n) = E^-_{ECC}(K_n)$.

$(ii)$ For the star graph $S_{1,n}$ with $n$ $(uv_r : 1 \leq r \leq n)$ pendant links, $\gamma_{ECC}(S_{1,n}) = 1 + \lceil \frac{n}{2} \rceil$. After the removal of the link, the graph becomes a $S_{1,n-1}$ graph and one isolated node.

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When $n$ is even, there is no change in $\chi_E$ and $\gamma_{ECC}(S_{1,n} - uv_r) = 1 + \lceil \frac{n}{2} \rceil$. When $n$ is odd, the value of $\chi_E$ is decreased by 1 and the center and isolated node have ordained the same color. So, the colors are all ordained maximum 2 nodes. In the process of choosing dominating nodes, we must choose the isolated node and suppose we choose the center node the number of dominating nodes in the color of center node is 2. Therefore, we must choose 2 nodes in all colors that implies we need to choose all nodes as a dominating nodes. Therefore, $\gamma_{ECC}(S_{1,n} - uv_r) = n + 1$. Hence, $uv_r \in \begin{cases} E^+_{ECC} & if\ n\ is\ odd \\ E^0_{ECC} & if\ n\ is\ even \end{cases}$.

**Remark 4.2:** There is a graph for which all the sets $E^-_{ECC}$, $E^0_{ECC}$ and $E^+_{ECC}$ are non-empty.

This is a general graph $G$, $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$. and $E(G) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$. $\chi_E(G) = 4$. ECC Dominating set $= \{v_1, v_2, v_5, v_6\}$ and $\gamma_{ECC}(G) = 4$.

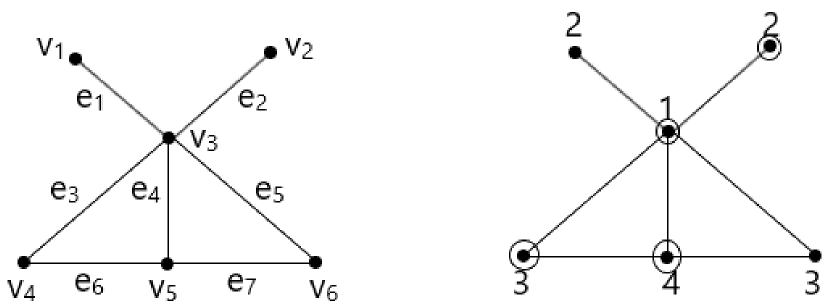


**Fig.4.1**

Now determine the node removal sets using the following figure.

$$\gamma_{ECC}(G - e_1) = 6 > \gamma_{ECC}(G)$$
$$\gamma_{ECC}(G - e_2) = 6 > \gamma_{ECC}(G)$$
$$\gamma_{ECC}(G - e_3) = 3 < \gamma_{ECC}(G)$$
$$\gamma_{ECC}(G - e_4) = 3 < \gamma_{ECC}(G)$$
$$\gamma_{ECC}(G - e_5) = 3 < \gamma_{ECC}(G)$$
$$\gamma_{ECC}(G - e_6) = 4 = \gamma_{ECC}(G)$$
$$\gamma_{ECC}(G - e_7) = 4 = \gamma_{ECC}(G)$$

Now conclude that $E^0_{ECC} = \{e_6, e_7\}$, $E^+_{ECC} = \{e_1, e_2\}$ and $E^-_{ECC} = \{e_3, e_4, e_5\}$. Hence, all the sets $E^-_{ECC}$, $E^0_{ECC}$ and $E^+_{ECC}$ are non-empty.

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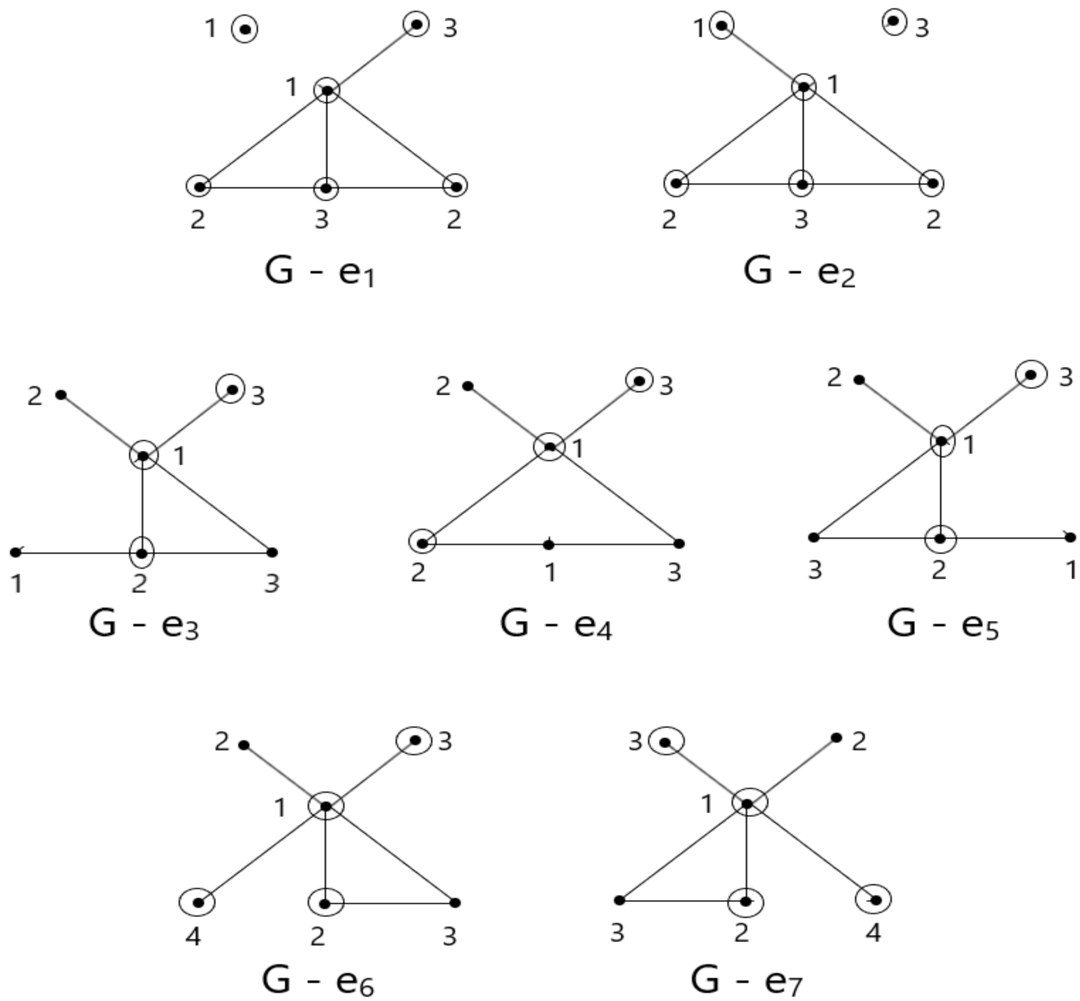


**Fig.4.2**

**Theorem 4.3:** For the path graph $n > 2$, $V(P_n) = (v_1, v_2, \dots, v_n)$ on $n$ nodes, and $E(P_n) = (e_1, e_2, \dots, e_{n-1})$ on $n-1$ links we have $1 \le r \le n-1$,

$$\gamma_{ECC}(P_n - e_r) = \begin{cases} \gamma_{ECC}(P_n) & if\ e_r \in E^0_{ECC} \\ \gamma_{ECC}(P_n) + 2 & if\ e_r \in E^+_{ECC} \end{cases}$$

1. If $n \equiv 0\ (mod\ 6)$

$$e_r \in \begin{cases} E^0_{ECC} & if\ r \equiv 0\ (mod\ 3) \\ E^+_{ECC} & otherwise \end{cases}$$

2. If $n \equiv -1 (mod\ 6)$

$$e_r \in \begin{cases} E^+_{ECC} & if\ r \equiv 1,4\ (mod\ 6) \\ E^0_{ECC} & otherwise \end{cases}$$

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3. If $n \equiv -2 (mod\ 6)$ $e_r \in \begin{cases} E_{ECC}^{+} & if\ r \equiv 1,3\ (mod\ 6) \\ E_{ECC}^{0} & otherwise \end{cases}$
4. Otherwise, $E(P_n) = E_{ECC}^0$

**Proof:** let $P_n$ be the path graph, $V(G) = \{v_r : 1 \leq r \leq n\}$ and $E(G) = \{e_r = v_r v_{r+1} : 1 \leq r \leq n-1\}$. $\chi_E(P_n) = 2$ and $\gamma_{ECC}(P_n) = 2\lceil \frac{n}{6} \rceil$ .

**Case 1:** If $n \equiv 0 (mod\ 6)$

Let $n = 6k$ for some $k \in \mathbb{N}$.

**Subcase 1:** If $r \equiv 0\ (mod\ 3)$

After the removal of the link, the path is divided into two paths with $3t$ and $n - 3t$ nodes $(1 \leq t \leq k)$. So, it has $t$ and $\gamma_{ECC}(P_n) - t$ dominating nodes. Therefore, $\gamma_{ECC}(P_n)$ is remained unchanged.

**Subcase 2:** Otherwise,

After the removal of the link, the path is divided into two paths with $t$ and $n - t$ nodes ($t \neq 3s\ and\ 1 \leq t \leq 3k-1, 1 \leq s \leq 2k-1$). When $t = 1\ and\ n-1$ the path is divided into $n - 1$ node path and an isolated node. So, it has $\gamma_{ECC}(P_n) + 1$ and $1$ dominating nodes. Otherwise, it has $\gamma_{ECC}(P_t)$ and $\gamma_{ECC}(P_{n-t})$ dominating nodes. Implies, $\gamma_{ECC}(P_t) + \gamma_{ECC}(P_{n-t}) = 2\lceil \frac{t}{6} \rceil + 2\lceil \frac{n-t}{6} \rceil$ . For the maximum value of $t$,

$$\gamma_{ECC}(P_t) + \gamma_{ECC}(P_{n-t}) = 2\left(\lceil \frac{3k-1}{6} \rceil + \lceil \frac{3k+1}{6} \rceil\right)$$

When $k$ is even, $\lceil \frac{3k-1}{6} \rceil + \lceil \frac{3k+1}{6} \rceil = \lceil \frac{3k}{6} \rceil + \lceil \frac{3k}{6} \rceil + 1 = \lceil \frac{6k}{6} \rceil + 1$. When $k$ is odd, $\lceil \frac{3k-1}{6} \rceil + \lceil \frac{3k+1}{6} \rceil = \lceil \frac{3k}{6} \rceil + \lceil \frac{3k}{6} \rceil = \lceil \frac{k}{2} \rceil + \lceil \frac{k}{2} \rceil = \frac{k+1}{2} + \frac{k+1}{2} = k + 1 = \lceil \frac{6k}{6} \rceil + 1$. So we have, $\gamma_{ECC}(P_t) + \gamma_{ECC}(P_{n-t}) = \gamma_{ECC}(P_n) + 2$. Therefore, $\gamma_{ECC}(P_n)$ is increased by 2.

Hence, $e_r \in \begin{cases} E_{ECC}^{0} & if\ r \equiv 0\ (mod\ 3) \\ E_{ECC}^{+} & otherwise \end{cases}$.

**Case 2:** If $n \equiv -1 (mod\ 6)$

Let $n = 6k - 1$ for some $k \in \mathbb{N}$. After the removal of the link, the path is divided into two paths with $r$ and $n - r$ nodes, one is odd path and another one is even path.

**Subcase 1:** If $r \equiv 1,4\ (mod\ 6)$

When $r = 1\ and\ n-1$ the path is divided into $n - 1$ node path and an isolated node. So, it has $\gamma_{ECC}(P_n) + 1$ and 1 dominating nodes. Otherwise, the path is divided into two paths with $3t + 1$ and $n - 3t - 1$ nodes $(0 \leq t \leq k-1)$. So, it has $\gamma_{ECC}(P_{3t+1})$ and $\gamma_{ECC}(P_{n-3t-1})$

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dominating nodes. Implies, $\gamma_{ECC}(P_{3t+1})+\gamma_{ECC}(P_{n-3t-1})=2\lceil\frac{3t+1}{6}\rceil+2\lceil\frac{n-3t-1}{6}\rceil$. For the maximum value of $t$,

$$\gamma_{ECC}(P_{3t+1})+\gamma_{ECC}(P_{n-3t-1})=2\left(\lceil\frac{3k-2}{6}\rceil+\lceil\frac{3k+1}{6}\rceil\right)$$

When $k$ is even, $\lceil\frac{3k-2}{6}\rceil+\lceil\frac{3k+1}{6}\rceil=\lceil\frac{3k}{6}\rceil+\lceil\frac{3k}{6}\rceil+1=\lceil\frac{6k}{6}\rceil+1$. When $k$ is odd, $\lceil\frac{3k-2}{6}\rceil+\lceil\frac{3k+1}{6}\rceil=\lceil\frac{3k}{6}\rceil+\lceil\frac{3k}{6}\rceil=\lceil\frac{k}{2}\rceil+\lceil\frac{k}{2}\rceil=\frac{k+1}{2}+\frac{k+1}{2}=k+1=\lceil\frac{6k}{6}\rceil+1=\lceil\frac{6k-1}{6}\rceil+1$. So we have, $\gamma_{ECC}(P_{3t+1})+\gamma_{ECC}(P_{n-3t-1})=\gamma_{ECC}(P_n)+2$. Therefore, $\gamma_{ECC}(P_n)$ is increased by 2.

**Subcase 2:** Otherwise,

Where $= 3t\ or\ 3t-1\ (1\le t\le 2k-1)$. We can be changed the coloring order according to preference in this case. So, it has $\gamma(P_r)$ and $\gamma(P_{n-r})$ dominating nodes. Implies, $\gamma(P_r)+\gamma(P_{n-r})=\lceil\frac{r}{3}\rceil+\lceil\frac{n-r}{3}\rceil$. For the maximum value of $t$,

When $r=3t$, $\gamma(P_{3t})+\gamma(P_{n-3t})=\lceil\frac{3(2k-1)}{3}\rceil+\lceil\frac{n-3(2k-1)}{3}\rceil=2k-1+\lceil\frac{6k-1-6k+3}{3}\rceil=2k=2\left(\frac{n+1}{6}\right)=2\lceil\frac{n}{6}\rceil$.

When $r=3t-1$, $\gamma(P_{3t-1})+\gamma(P_{n-3t+1})=\lceil\frac{3(2k-1)-1}{3}\rceil+\lceil\frac{n-3(2k-1)+1}{3}\rceil=\lceil\frac{6k-4}{3}\rceil+1=\lceil\frac{6k-4}{6}\rceil+k-1+1=2k=2\left(\frac{n+1}{6}\right)=2\lceil\frac{n}{6}\rceil$.. So we have, $\gamma(P_r)+\gamma(P_{n-r})=\gamma_{ECC}(P_n)$. Therefore, $\gamma_{ECC}(P_n)$ is remained unchanged.

Hence, $e_r\in\begin{cases}E_{ECC}^{+} & if\ r\equiv 1,4\ (mod\ 6)\\ E_{ECC}^{0} & otherwise\end{cases}$.

**Case 3:** If $n\equiv -2(mod\ 6)$

Let $n=6k-2$ for some $k\in\mathbb{N}$. After the removal of the link, the path is divided into two paths with $r$ and $n-r$ nodes, both is either odd or even path.

**Subcase 1:** If $r\equiv 1\ (mod\ 6)$

When $r=1$ the path is divided into $n-1$ node path and an isolated node. So, it has $\gamma_{ECC}(P_n)+1$ and 1 dominating nodes. Otherwise, the path is divided into two paths with $6t+1$ and $n-6t-1$ nodes $(1\le t\le k-1)$. So, it has $\gamma_{ECC}(P_{6t+1})$ and $\gamma_{ECC}(P_{n-6t-1})$ dominating nodes. Implies, $\gamma_{ECC}(P_{6t+1})+\gamma_{ECC}(P_{n-6t-1})=2\lceil\frac{6t+1}{6}\rceil+2\lceil\frac{n-6t-1}{6}\rceil$. For the maximum value of $t$,

$$\gamma_{ECC}(P_{6t+1})+\gamma_{ECC}(P_{n-6t-1})=2\left(\lceil\frac{6k-5}{6}\rceil+1\right)=2\left(\lceil\frac{6k-2}{6}\rceil+1\right)$$

$$=2\left(\lceil\frac{n}{6}\rceil+1\right)=\gamma_{ECC}(P_n)+2.$$

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**Subcase 2:** If $r \equiv 3\ (mod\ 6)$

When $r = n-1$ the path is divided into $n-1$ node path and an isolated node. So, it has $\gamma_{ECC}(P_n)+1$ and 1 dominating nodes. Otherwise, the path is divided into two paths with $6t+3$ and $n-6t-3$ nodes $(0 \le t \le k-2, k > 1)$. So, it has $\gamma_{ECC}(P_{6t+3})$ and $\gamma_{ECC}(P_{n-6t-3})$ dominating nodes. Implies, $\gamma_{ECC}(P_{6t+3})+\gamma_{ECC}(P_{n-6t-3}) = 2\lceil \frac{6t+3}{6} \rceil + 2\lceil \frac{n-6t-3}{6} \rceil$. For the maximum value of $t$,

$$\gamma_{ECC}(P_{6t+3})+\gamma_{ECC}(P_{n-6t-3}) = 2\left(\lceil \frac{6k-9}{6} \rceil + 2\right) = 2\left(\lceil \frac{6k-2}{6} \rceil - 1 + 2\right)$$

$$= 2\left(\lceil \frac{n}{6} \rceil + 1\right) = \gamma_{ECC}(P_n) + 2.$$

**Subcase 3:** Otherwise,

Where $= 2t\ (1 \le t \le 3k-2)\ or\ 6t-1\ \ (1 \le t \le k-1\,, k > 1)$. We can be changed the coloring order according to preference in this case. So, it has $\gamma(P_r)$ and $\gamma(P_{n-r})$ dominating nodes. Implies, $\gamma(P_r)+\gamma(P_{n-r}) = \lceil \frac{r}{3} \rceil + \lceil \frac{n-r}{3} \rceil$. For the maximum value of $t$,

When $r = 2t$, $\gamma(P_{2t})+\gamma(P_{n-2t}) = \lceil \frac{2(3k-2)}{3} \rceil + \lceil \frac{n-2(3k-2)}{3} \rceil = \lceil \frac{6k-4}{3} \rceil + \lceil \frac{6k-2-6k+4}{3} \rceil = 2k-1+1 = 2k = 2\left(\frac{n+2}{6}\right) = 2\lceil \frac{n}{6} \rceil$.

When $r = 6t-1$, $\gamma(P_{6t-1})+\gamma(P_{n-6t+1}) = \lceil \frac{6(k-1)-1}{3} \rceil + \lceil \frac{n-6(k-1)+1}{3} \rceil = \lceil \frac{6k-7}{3} \rceil + \lceil \frac{6k-2-6k+6+1}{3} \rceil = 2k-2+2 = 2k = 2\left(\frac{n+2}{6}\right) = 2\lceil \frac{n}{6} \rceil$. So we have, $\gamma(P_r)+\gamma(P_{n-r}) = \gamma_{ECC}(P_n)$. Therefore, $\gamma_{ECC}(P_n)$ is remained unchanged.

Hence, $e_r \in \begin{cases} E_{ECC}^{+} & if\ r \equiv 1,3\ (mod\ 6) \\ E_{ECC}^{0} & otherwise \end{cases}$.

**Case 4:** Otherwise

**Subcase 1:** If $n \equiv -3(mod\ 6)$

Let $n = 6k-3$ for some $k \in \mathbb{N}$. After the removal of the link, the path is divided into two paths with $r$ and $n-r$ nodes $(1 \le r \le n-1)$, when $r = 1$ the path is divided into one path and one isolated node. If $r \equiv 0\ (mod\ 3)$ the path is divided into two paths with $3t$ and $n-3t$ nodes $(1 \le t \le 2k-2)$. So, it has $\gamma_{ECC}(P_{3t})$ and $\gamma_{ECC}(P_{n-3t})$ dominating nodes. Implies, $\gamma_{ECC}(P_{3t})+\gamma_{ECC}(P_{n-3t}) = 2\lceil \frac{3t}{6} \rceil + 2\lceil \frac{n-3t}{6} \rceil$. For the maximum value of $t$,

$$\gamma_{ECC}(P_{3t})+\gamma_{ECC}(P_{n-3t}) = 2(k-1+1) = 2(k) = 2\left(\frac{n+3}{6}\right)$$

$$= 2\left(\lceil \frac{n}{6} \rceil\right) = \gamma_{ECC}(P_n).$$

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If $r \not\equiv 0\ (mod\ 3)$ the path is divided into two paths with $t$ and $n-t$ nodes $(1 \le t \le 6k-4, t \not\equiv 0\ (mod\ 3))$. So, it has $\gamma(P_t)$ and $\gamma(P_{n-t})$ dominating nodes. Implies, $\gamma(P_t) + \gamma(P_{n-t}) = \lceil \frac{t}{3} \rceil + \lceil \frac{n-t}{3} \rceil$. For the maximum value of $t$,

$$\gamma(P_t) + \gamma(P_{n-t}) = 2k - 1 + 1 = 2(k) = 2\left(\frac{n+3}{6}\right)$$

$$= 2\left(\lceil \frac{n}{6} \rceil\right) = \gamma_{ECC}(P_n).$$

Hence, $E(P_n) = E^0_{ECC}$

**Subcase 2:** If $n \equiv 2(mod\ 6)$

Let $n = 6k + 2$ for some $k \in \mathbb{N}$. After the removal of the link, the path is divided into two paths with $r$ and $n-r$ nodes $(1 \le r \le n-1)$, when $r = 1$ the path is divided into one path and one isolated node. So, it has $1$ and $\gamma(P_{n-1})$ dominating nodes. Implies, $1 + \gamma(P_{n-1}) = 1 + \lceil \frac{n-1}{3} \rceil = 2(k+1) = 2\left(\frac{n+4}{6}\right) = 2\lceil \frac{n}{6} \rceil = \gamma_{ECC}(P_n)$. Otherwise, the path is divided into two paths with $t$ and $n-t$ nodes $(2 \le t \le 6k)$. So, it has $\gamma_{ECC}(P_t)$ and $\gamma_{ECC}(P_{n-t})$ dominating nodes. Implies, $\gamma_{ECC}(P_t) + \gamma_{ECC}(P_{n-t}) = 2\lceil \frac{t}{6} \rceil + 2\lceil \frac{n-t}{6} \rceil$. For the maximum value of $t$,

$$\gamma_{ECC}(P_t) + \gamma_{ECC}(P_{n-t}) = 2(k+1) = 2\left(\frac{n+4}{6}\right)$$

$$= 2\left(\lceil \frac{n}{6} \rceil\right) = \gamma_{ECC}(P_n).$$

Hence, $E(P_n) = E^0_{ECC}$

**Subcase 3:** If $n \equiv 1(mod\ 6)$

Let $n = 6k + 1$ for some $k \in \mathbb{N}$. After the removal of the link, the path is divided into two paths with $r$ and $n-r$ nodes $(1 \le r \le n-1)$, when $r = 1$ the path is divided into one path and one isolated node. So, it has $1$ and $\gamma_{ECC}(P_{n-1}) + 1$ dominating nodes. Implies, $2 + \gamma_{ECC}(P_{n-1}) = 2 + 2\lceil \frac{n-1}{6} \rceil = 2(k+1) = 2\left(\frac{n+5}{6}\right) = 2\lceil \frac{n}{6} \rceil = \gamma_{ECC}(P_n)$. Otherwise, the path is divided into two paths with $t$ and $n-t$ nodes $(2 \le t \le 6k-1)$. So, it has $\gamma_{ECC}(P_t)$ and $\gamma_{ECC}(P_{n-t})$ dominating nodes. Implies, $\gamma_{ECC}(P_t) + \gamma_{ECC}(P_{n-t}) = 2\lceil \frac{t}{6} \rceil + 2\lceil \frac{n-t}{6} \rceil$. For the maximum value of $t$,

$$\gamma_{ECC}(P_t) + \gamma_{ECC}(P_{n-t}) = 2\lceil \frac{6k-1}{6} \rceil + 2\lceil \frac{n-6k+1}{6} \rceil = 2(k+1) = 2\left(\frac{n+5}{6}\right)$$

$$= 2\left(\lceil \frac{n}{6} \rceil\right) = \gamma_{ECC}(P_n).$$

Hence, $E(P_n) = E^0_{ECC}$

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**Theorem 4.4:** For the cycle graph $n > 2$, $C_n = (v_1, v_2, \dots, v_n)$ on $n$ nodes, we have $1 \leq r \leq n$, $1 \leq t \leq 3$, $\gamma_{ECC}(C_n - v_r) = \begin{cases} \gamma_{ECC}(C_n) - t & if\ V = V_{ECC}^{-} \\ \gamma_{ECC}(C_n) & if\ V = V_{ECC}^{0} \\ \gamma_{ECC}(C_n) + 1 & if\ V = V_{ECC}^{+} \end{cases}$

1. If $n$ is even, $V = V_{ECC}^{o}$
2. If $n$ is odd and
   I. If $n \equiv -3, -1\ (mod\ 18)$, $V = V_{ECC}^{o}$
   II. If $n \equiv 0\ (mod\ 9)$, $V = V_{ECC}^{+}$
   III. If $n \equiv 3,5,7\ (mod\ 18)$, $V = V_{ECC}^{-}$ and $t = 1$
   IV. If $n \equiv -7, -5\ (mod\ 18)$, $V = V_{ECC}^{-}$ and $t = 2$
   V. If $n \equiv 1\ (mod\ 18)$, $V = V_{ECC}^{-}$ and $t = 3$

**Proof:** let $C_n$ be the cycle graph, $V(G) = \{v_r : 1 \leq r \leq n\}$ and $E(G) = \{v_r v_{r+1}, v_n v_1 : 1 \leq r \leq n-1\}$. $\chi_E(C_n) = \begin{cases} 2\ if\ n\ is\ even \\ 3\ if\ n\ is\ odd \end{cases}$ and $\gamma_{ECC}(C_n) = \begin{cases} 2 \lceil \frac{n}{6} \rceil\ if\ n\ is\ even \\ 3 \lceil \frac{n}{9} \rceil\ if\ n\ is\ odd \end{cases}$. Now removal of the vertex $V(C_n)$ the cycle is changed to a path with $n-1$ nodes.

**Case 1:** $n$ is even

When $n$ is even $\gamma_{ECC}(C_n) = \gamma_{ECC}(P_n)$ and $\gamma_{ECC}(P_n) = \gamma_{ECC}(P_{n-1})$ which implies $\gamma_{ECC}(C_n) = \gamma_{ECC}(P_{n-1})$. Therefore, $\gamma_{ECC}(C_n - v_r)$ remains unaltered. Hence, $V(C_n) = V_{ECC}^{0}$.

**Case 2:** $n$ is odd

When $n$ is odd, after the removal of one node in an odd cycle it becomes an even path.

**Sub Case 1:** $n \equiv -3, -1\ (mod\ 18)$

Let $n = 18k - 3\ and\ 18k - 1$, $k$ be a natural number. When $n \equiv -3, -1\ (mod\ 18)$, $\gamma_{ECC}(C_n) = \gamma_{ECC}(P_n)$ and $\gamma_{ECC}(P_n) = \gamma_{ECC}(P_{n-1})$ which implies $\gamma_{ECC}(C_n) = \gamma_{ECC}(P_{n-1})$. Therefore, $\gamma_{ECC}(C_n - v_r)$ remains unaltered. Hence, $V(C_n) = V_{ECC}^{0}$.

**Sub Case 2:** $n \equiv 0\ (mod\ 9)$

Let $n = 9k$, $k$ be a natural number. When $n \equiv 0\ (mod\ 9)$, $\gamma_{ECC}(C_n) = \gamma_{ECC}(P_n) - 1$ and $\gamma_{ECC}(P_n) = \gamma_{ECC}(P_{n-1})$ which implies $\gamma_{ECC}(C_n) = \gamma_{ECC}(P_{n-1}) - 1$. Therefore, $\gamma_{ECC}(C_n - v_r)$ is increased by 1. Hence, $V(C_n) = V_{ECC}^{+}$.

**Sub Case 3:** $n \equiv 3,5,7\ (mod\ 18)$

Let $n = 18k + 3, 18k + 5\ and\ 18k + 7$, $k$ be a whole number. When $n \equiv 3,5\ (mod\ 18)$, $\gamma_{ECC}(C_n) = \gamma_{ECC}(P_n) + 1$ and $\gamma_{ECC}(P_n) = \gamma_{ECC}(P_{n-1})$ which implies $\gamma_{ECC}(C_n) = \gamma_{ECC}(P_{n-1}) + 1$. When $n \equiv 7\ (mod\ 18)$, $\gamma_{ECC}(C_n) = \gamma_{ECC}(P_n) - 1$ and $\gamma_{ECC}(P_n) =$

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$\gamma_{ECC}(P_{n-1}) + 2$ which implies $\gamma_{ECC}(C_n) = \gamma_{ECC}(P_{n-1}) + 1$. Therefore, $\gamma_{ECC}(C_n - v_r)$ is decreased by 1. Hence, $V(C_n) = V_{ECC}^{-}$.

**Sub Case 4:** $n \equiv -7, -5\ (mod\ 18)$

Let $n = 18k - 7\ and\ 18k - 5$, $k$ be a natural number. When $n \equiv -7\ (mod\ 18)$, $\gamma_{ECC}(C_n) = \gamma_{ECC}(P_n) + 2$ and $\gamma_{ECC}(P_n) = \gamma_{ECC}(P_{n-1})$ which implies $\gamma_{ECC}(C_n) = \gamma_{ECC}(P_{n-1}) + 2$. When $n \equiv -5\ (mod\ 18)$, $\gamma_{ECC}(C_n) = \gamma_{ECC}(P_n)$ and $\gamma_{ECC}(P_n) = \gamma_{ECC}(P_{n-1}) + 2$ which implies $\gamma_{ECC}(C_n) = \gamma_{ECC}(P_{n-1}) + 2$. Therefore, $\gamma_{ECC}(C_n - v_r)$ is decreased by 2. Hence, $V(C_n) = V_{ECC}^{-}$.

**Sub Case 5:** $n \equiv 1\ (mod\ 18)$

Let $n = 18k + 1$, $k$ be a natural number. When $n \equiv 1\ (mod\ 18)$, $\gamma_{ECC}(C_n) = \gamma_{ECC}(P_n) + 1$ and $\gamma_{ECC}(P_n) = \gamma_{ECC}(P_{n-1}) + 2$ which implies $\gamma_{ECC}(C_n) = \gamma_{ECC}(P_{n-1}) + 3$. Therefore, $\gamma_{ECC}(C_n - v_r)$ is decreased by 3. Hence, $V(C_n) = V_{ECC}^{+}$.

## CONCLUSION

The study of the effect of the removal of a link in any graph theoretic parameter has interesting applications in the context of the network. In this paper, a similar study has been initiated concerning the Equitable Color Class Domination number for a graph $G$.

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