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MEAN D-DISTANCE IN GRAPHS

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Abstract

The d-distance between vertices of a graph is obtained by considering the path lengths and as well as the degrees of vertices present on the path. The mean distance between the vertices of a connected graph is the average of the distances between all pairs of vertices of the graph. In this article we study the mean d-distance between the vertices of a graph.2000 Mathematics Subject Classification. 05C12.

Keywords and Phrases: D-Distance, Mean D-Distance, Diameter

1. INTRODUCTION

The concept of distance is one of the important concepts in study of graphs. It is used in isomorphism testing, graph operations, hamiltonicity problems, extremal problems on connectivity and diameter, convexity in graphs etc. Distance is the basis of many concepts of symmetry in graphs. In addition to the usual distance, $d(u,v)$ between two vertices $u,v \in V(G)$ we have detour distance (introduced by Chartrand et al, see [2]), superior distance (introduced by Kathiresan and Marimuthu, see [5]), signal distance (introduced by Kathiresan and Sumathi, see [6]), degree distance etc. In an earlier article [8], the authors, Reddy Babu and Varma introduced the concept of D-distance between vertices of a graph G by considering not only path length between vertices, but also the degrees of all vertices present in a path while defining the D-distance. The concept of d-distance introduced by see [11].

The article is arranged as follows. In study some properties of mean d- distance and in $3, we calculate the average D-distance between n vertices for some classes of graphs throughout this article, by a graph $G(V, E)$ or simply $G,$ we mean a non-trivial finite, undirected graph without multiple edges and loops. Further all graphs we consider are

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connected. The d-distance, dd (u, v), between two vertices u, v of a connected graph G is defined as $d^d(u,v) = \min\{d(u,v) + \deg(u) + \deg(v) + \deg(u)\deg(v)\}$. Similarly, we can define the mean d-distance of a graph as Let G be a connected graph of order n. The mean d-distance between vertices of G denoted by μ^d, is defined as $\mu^d(G) = (n_{C_2})^{-1} d^d(u,v)$ where $d^d(u,v)$ denotes the d-distance between the vertices u and v . The total d-distance $[TdD]$ of graph G is the number given by $\sum d^d(u,v)$.

2. MEAN D-DISTANCE

In this section we prove some results on mean d-distance between vertices. We begin with a theorem which connects the number of vertices and mean d-distance. This leads to some more results.

Theorem 2.1. Let G1 and G2 be two connected graphs having same number edges and same diameters. If the number of vertices in G1 is more than the number of vertices in G2 then mean d-distance of G1 is more than mean d-distance of G2.

Proof: Since the diameters of these two graphs are the same, the largest entries in the d-distance matrix of these graphs are the same. The number of the pairs of vertices is more in G_1 and hence total d-distance value of G1 is more. Since number of edges in G_1 and G_2 are same. This implies the mean d-distance of G_1 is more than mean d-distance of G_2

Theorem 2.2. Let G1 and G2 be two connected graphs of same number of edges and $diam(G_1) < diam(G_2)$. Then μd (G1) > μd (G2).

Proof. Since G1, G2 have same number of edges and $diam(G_1) < diam(G_2)$, it is clear that $V(G_1) > V(G_2)$. Then by above theorem, we have $\mu^d(G_1) > \mu^d(G_2)$.

Theorem 2.3. Let G1 and G2 be two connected graphs having same number of edges and diameters. If $\delta(G_1) < \delta(G_2)$ then $\mu^d(G_1) > \mu^d(G_2)$.

Proof. If $\delta(G_1) < \delta(G_2)$ implies V (G1) > V (G2). Then by theorem 1, μd (G1) > μd (G2).

Theorem 2. 4. Let G1 and G2 be two connected graphs having same number of edges and same diameters. If $\delta^1(G_1) < \delta^1(G_2)$ G2) then $\mu^d(G_1) > \mu^d(G_2)$.

Proof. Since $\delta^1(G_1) < \delta^1(G_2)$ we have $V(G_1) > V(G_2)$. Then by theorem1, $\mu^d(G_1) > \mu^d(G_2)$.

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3. RESULTS ON SOME CLASSES OF GRAPHS

Here we calculate the mean d-distance for some classes of graphs.

Theorem 3.1. The mean d-distance of complete graph K_n is $\mu^d(K_n) = 2n^2$.

Proof. Every vertex taken from K_n has $n-1$ vertex neighbors. The d-distance between any vertex and its neighbors is n^2. Thus the total d-distance $TdD = (n_{C_2})(n^2)$ and hence

$$\mu^d(K_n) = \frac{TdD}{(n_{C_2})} = 2n^2.$$

Theorem 3.2. The mean d-distance of complete bipartite graph $K_{m,n}$ is $\mu^d(K_{m,n}) = 4\frac{(m_{C_2})(n^2+2n+2)+(n_{C_2})(m^2+2m+2)+3mn(n+1)}{(m+n)(m+n-1)}$.

Proof. Let $V(K_{m,n}) = X \cup Y$ where X = {a1, a2, a3, · · ·, an} and
Y = {b1, b2, b3, · · ·, bn}.

$d^d(a_{i,} a_j) = n^2+2n+2$, $d^d(b_{i,} b_j) = m^2+2m+2$ and $d^d(a_i, b_j) = 3(n+1).$ thus total d-distance

$$TdD = 2(m_{C_2})(n^2+2n+2)+(n_{C_2})(m^2+2m+2)+3mn(n+1)$$

$$\mu^d(K_{m,n}) = 2\frac{(m_{C_2})(n^2+2n+2)+(n_{C_2})(m^2+2m+2)+3mn(n+1)}{((m+n+1)_{C_2})}$$

$$= 4\frac{(m_{C_2})(n^2+2n+2)+(n_{C_2})(m^2+2m+2)+3mn(n+1)}{(m+n)(m+n-1)}.$$

Theorem 3.3. The mean d-distance of star graph St1, n is given by $\mu^d(St_{1,n}) = \frac{2(9n-1)}{n-1}$.

Proof. In Star graph there are $n+1$ vertices. For the central vertex there are n neighbor vertices and all other vertices has only one neighbor vertex, namely the central vertex. For the central vertex there are no distinct vertices, whereas all others have $n-1$ distinct vertices. The d-distance between any vertex and its distinct vertex is $4n(n+1)$, finally $TdD = n(9n-1)$ then $\mu^d(st_{1,n}) = \frac{TdD}{n_{C_2}} = \frac{2(9n-1)}{n-1}.$

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