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SOLVING AIRLINE CREW ASSIGNMENT PROBLEM BY USING COLUMN PENALTY METHOD

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Abstract

Crew scheduling problem (CSP) is among the hardest optimization problems where crew costs are the second highest expenses for airlines. This makes it a crucial factor for an airline's survival. The scheduling process is composed of the Crew Pairing Problem (CPP) and the Crew Assignment Problem (CAP). A new approach to address the airline crew assignment problem will be comprehensively explored in this research paper. The purpose of this article is to present an effective means of optimizing crew pairings that would reduce expenses and time away from the base of crews. This model through the use of Column Penalty Method (CPM), uses less required matrix order, iteration and number of tables as well as the steps involved in the problem presents an efficient optimal solution. This approach consists of mainly two stages: generating crew pairings and optimization phase. The data have been collected from the airline companies that shows that the optimal solution obtained by Column Penalty Method is better than the existing methods.

Keywords: Airline Crew Assignment Problem, Column Penalty Method, Crew Scheduling Problem, Optimal Solution.

1. INTRODUCTION

How can the cheapest way of allocating tasks to operators be determined? An example of these problems is the Airline Crew Assignment Problem (ACAP), also known as crew scheduling problem, which is a real-world case.

The airline industry has extensive planning and scheduling requirements. As a highly competitive industry, managing operating costs is very critical. Since 1950s, airlines have been using operations research techniques to ensure that their resources are used efficiently and they meet the demand (Anbil, 1991).

From an economic standpoint, Crew Assignment Problem (CAP) for airlines becomes a critical concern due to market competition. Direct operating expenses are mainly composed of crew costs that follow fixed aircraft expenses and fuel prices. Therefore,

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significant cost savings may be achieved by resolving the Crew Assignment Problem optimally.

This paper explains how to find an optimal solution for the crew assignment (rostering) problem by constraints set, i.e. all flight legs cover and obtaining minimal cost crew pairings set. Solutions for this problem are obtained by breaking it down into crew assignment (rostering) problem and crew pairing problem. First, there is a need to get pairing of the flight legs that depart and arrive in the similar city i.e. crew base to find the solution of the crew pairing problem. Then, optimize the assignment problem of crews where the flight legs set are assigned to the pairings that are having particular members of crews.

The ACAP involves assigning crews of flights to bases while taking into account certain operational constraints and optimally satisfying some objectives functions. The aircrew assignment problem aims at finding an optimal minimum cost assignment for flight crews with minimum time away from their home base. This depends on various factors including flying hours per month for each crew member, the duty day total hours, returning trip back to base and rest time (Barnhart, 1997).

Ranga (1998) developed the Sprint method for large scale crew scheduling, Gopalakrishnan (2005) discusses and surveys the state-of-the-art different approaches for solving the crew scheduling problem, Sumarti (2017) used Simple Fuzzy Logic Approach to solve ACAP, Radu (2018) proposed Robust Airline Crew Pairing Optimization for Short-Haul Flight, Novianingsih (2018) **)** solved airline crew pairing problem by a new heuristic method, and Quesnel (2019) **)** developed a new two-phase approach for the aircrew scheduling problem that takes into account crew qualifications and language requirements during pairing generation, Mallick. C (2023) used assignment Technique to minimize the airline crew scheduling problem, Guo. C (2024) proposed a method on optimizing of flight crew scheduling problem (CSP) considering pilot fatigue. All of these methods cannot always guarantee optimal solutions and most are complex. Unlike those used in the past, CPM strives to achieve an optimal solution with fewer steps and high efficiency that minimize waiting time away from base cities for crews thereby improving operational efficiency.

2. OPERATIONAL DEFINITIONS

Before delving into the problem, certain terminologies need to be explained properly. For instance, a segment or flight leg means one plane trip without any stops in between. A duty time refers to a part of the crew’s working day which entails several flight legs spaced by some time off, also called “sits.” Briefing marks the start of the duty while debriefing marks its end. On the other hand, a pairing constitutes several consecutive duty periods starting from and going back to the same base where crews are stationed – it also includes overnight rests between each duty. The crew pairing problem relates to scheduling flights over different time frames such as daily, weekly or monthly. (El-Habashy, 2014) provided the timetable in this case usually covers a month and includes

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several flight frequencies organized within different schedules. All flights are assumed to be running every day in case of daily version of the problem.

3. MATHEMATICAL MODEL

The problem of assigning crews in the airline industry pertains to the creation of a wide range of pairings that encompass all planned flights at the minimum cost possible. This dilemma is represented as Set Covering Problem (SCP) and Set Partition Problem (SPP). The SCP is different from SPP in a way that SPP requires each element to appear only once, whereas every element in an SCP must appear in at least one subset. Crew assignments and pairing optimization often favor the set covering model instead of a set partition model otherwise it would usually arrive at an unfeasible solution due to the requirement of over-coverage in actual schedules, which will make partitions not feasible.

The input data for this problem is represented by a *S* matrix, where columns correspond to pairings where rows correspond to flights. During the step of optimization, a subset of pairings is selected from *P* that is all possible pairings to ensure that all schedules of flights (*f*) are covered while minimizing the periods of rest. Define x_{lk} for every $k \in P$ as

$$x_{lk} = \begin{cases} 1, & \textit{as long as } \mathbf{k} \textit{ pairing used.} \\ 0, & \textit{else} \end{cases}$$

Allow Ω_k represents ***k*** *pairing* expense or *cost*

And ***f*** represent the all flights sets that need to be covered.

For each $k \in \boldsymbol{f}$ allow P^k represents all pairings sets which covers ***N*** flight

$f= \{f_1, f_2, \ldots, _N\}$

3.1. Creation of pairings problem can be written as follows:

Let;

Min

$$S = \sum_{k \in P} \Omega_k \, x_k \qquad \ldots (3.1.1)$$

Subject to

$$\sum_{k \in P^l} x_k = 1 \qquad \forall \; l \in F \; \ldots (3.1.2)$$

$$X_k \in \{0,1\} \qquad \forall \; k \in P \qquad \ldots (3.1.3)$$

$$P = \{P_1, P_2, \ldots, P_m\}$$

Aiming to minimize the total cost of the pairings, the objective function (3.1.1) is to be utilized. Exactly one time covering for each flight leg in the pairings is ensured by

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constraints (3.1.2), and pairing variables are constrained to be binary by constraints (3.1.3).

Let $S = (a_{lk} : l \in f, k \in P)$ represent the 0,1 pairing of flights incidence matrix

as

$P = \{ P_1, P_2, \ldots, P_m\}$ *and*

$$a_{lk} = \begin{cases} 1, & \text{as long as pairing } \mathbf{k} \text{ contain flight } \mathbf{l}. \\ 0, & \text{Else} \end{cases}$$

Next this may be expressed as Set Covering Problem (SCP)

Min

$$\sum_{k\in P} \Omega_k \, x_k$$

Conditional

$$S = \sum_{k\in P} a_{lk} \qquad x_l \ \geq 1 \qquad \forall\, l \ \in f \qquad \ldots (3.1.4)$$

$$X_l \in \{0,1\}$$

As ***S*** represents table or matrix having flights rows and pairings columns.

3.2. For Rest time calculation (Time among air travel or flights):

Allow $\boldsymbol{R_{lk}}$ represents Rest time among flights within pairing.

$\boldsymbol{A_t}$ = Arrival timing,

$\boldsymbol{D_t}$ = Departure timing,

N = flights number in corresponding paring ***k***.

$\forall \quad f \subseteq P \qquad l \in 1,\ldots, N \quad$ and $\quad k \in 1,..,P \qquad \forall\, P \in Base$, $P = \{ P_1, P_2, \ldots, P_m\}$

$$R_{lk} = [(\sum_{l=1}^{n} At_1 - \sum_{l=1}^{n} Dt_2) + \cdots + (\sum_{l=1}^{n} At_{n-1} - \sum_{l=1}^{n-1} Dt_n)]_k \qquad \ldots (3.2.1)$$

$$\text{As} \quad \Omega_{lk} = \begin{cases} R_{lk}, & \forall\, R_{lk} \in P \in Base \\ 0, & \text{Else} \end{cases}$$

$$R_{lk} = \{ R_{11}, R_{12}, \ldots R_{Nm} \}$$

Min

$$S = \sum_{l=1} \sum_{k=1} \Omega_{lk} \, x_{lk} \qquad \ldots (3.2.2)$$

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Conditional

$l \in 1,2,\ldots,f$ and $k \in 1,2,..,P$

$$x_{lk} = \begin{cases} 1, & \textit{as long as k pairing considered.} \\ 0, & \textit{Otherwise} \end{cases}$$

3.3. Solving Assignment Problem by using Column Penalty Method (CPM)

Min

$$S = \sum_{l=1}^{N} \sum_{k=1}^{N} \Omega_{lk}\, x_{lk} \qquad \ldots (3.3.1)$$

Conditional

$$q_{lk} = \sum_{l=1}^{N} \beta_{lk} - \sum_{k=1}^{N} \alpha_{lk} \qquad \ldots (3.3.2)$$

As $\beta_{lk} \geq \alpha_{lk} > 0, \quad x_{lk} = 1$

Denote

α_{lk} = minimal value in each row, $l, k = 1,2,..,N$

β_{lk}= maximal value in corresponding column,

q_{lk} = highest difference betweenβ_{lk} andα_{lk} , corresponding column,

$$\Omega_{lk} = \begin{cases} q_{lk} & , q_{lk} > 0 \\ 0, & \text{Else} \end{cases}$$

4. COST COMPUTING

Pairing costs include an approximation of crew's pay as well as other expenses such as hotel charges.

For flights, the assignment cost of crew is arrived at using several calculations. According to Air Blue Airlines and PIA (Pakistan International Airlines), cockpit crew gets fixed daily wages of Rs. 10,000/- (around $65) while for a cabin crew it’s only Rs. 2,000/- (about $15). Additionally, they are paid in accordance with the flying time whereby cockpit crew earns Rs. 5,000/- per hour (equivalent to $35) and for a cabin crew member he/she would be getting Rs. 2,000/- ($15) every hour. Money is also paid out as compensation during sleep overs and layover periods between flights with $120 meant for cockpit crew per night and $60 meant for cabin crew. Thus the duration serves as an estimate of flight cost per person. The cost of pairing (*Cd*) in hours can be determined using two values; TAFB and *Cd* (the sum of individual duty charges).

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The expression for the pairing cost is as follows:

C_p = *maxi {Time Away from Base (TAFB),* $\sum_{k\in P} C_d$} …… (4.1)

Cp the cost of a pairing or expenses is defined as the differentiation among the crew time spends away from base and the actual time in the air. In this study, the expenses of a pairing is basically calculated based on the hours that the crew spends without flying, excluding time on the air.

C_p = *maxi* {*Time Away from Base (TAFB)* – Time on the air} …..(4.2)

4.1. Roundtrip Flights Cases

- **Firstly, when flight leg be among couple of cities *M to N and N to M***

For optimizing the assignments,

1) Compute rest time (layover times) as per tabled information (timing and city of takeoff and landing). It is contingent upon airline policies.
2) For each city create schedule as a base.
3) From two corresponding cell chose the minimum value.
4) Use Column Penalty Method (CPM) (**R. M. Ahmed et.al. 2020)** to assign members of flights crews to the city base with minimal rest time.
5) Use the Hungarian method for demonstration of the solution.

- **Secondly, when flight trips are among more than couple of cities *M, N, K*…**

1) Discover the flights pairing as per information in every city as a base (timing and city of takeoff and landing) it is contingent upon airline policies.

   (Where first flight destination and the second origin must match)
2) In every pairing compute rest time. (Time among flight trips)
3) Use Column Penalty Method to allocate flights to pairings and identify the minimal rest time, then determine crew base to reduce the total expenses or cost.

Table 4.1.1: Represents the formulation of the problem matrix

	Pairings			
	C_1	C_2	…	C_n
Flights	P_1	P_2	…	P_k
f_1	R_{11}	R_{12}	…	R_{1k}
f_2	R_{21}	R_{22}	…	R_{2k}
⋮	⋮	⋮	⋮	⋮
f_l	R_{l1}	R_{l2}	…	R_{lk}

Where; P_k **=** Flight Pairing**, Rlk =** Rest time**,** C_n**=** Crews**.**

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5. COLUMN PENALTY METHOD (CPM) ALGORITHM FOR THE ASSIGNMENT PROBLEM OF AIRLINE CREW

Conceder f_1 , $f_2, \ldots, f_l$ as flights, where pairings represents as $P_1, P_2, \ldots, P_k$, every flight pairing has specific crew members $C_1, C_2, \cdots, C_k$. Then, apply the upcoming steps for optimizing the assignment problem.

Step 1: Construct expenses or cost platform for crew assignment problem by representing flights in rows and pairings in columns.

	C_1	C_2	⋯	C_k
	P_1	P_2	⋯	P_K
f_1				
f_2				
⋮				
f_l				

Step 2: Then, create couple of columns, the left for flight trips, and the other one for pairing.

1st Column	2nd Column 2
⋮	⋮
f_l	P_k
⋮	⋮

Step 3: In flight column, write the flights, for instance $f_1, f_2, \ldots, f_l$. then for each flight locate minimal rest time, wherever minima value is appear in the respecting pairing, then select it and attach it in the pairings column. Repeat the technique for each flight legs.

1st Column	2nd Column
f_1	R_{min}
f_2	⋮
⋮	⋮
f_l	

R_{nim} (Minimal rest time),

Step 4: If pairing is unique for the flight (where in this pairing there is no other flight has a shorter rest time), then locate that flight to the corresponding pairing.

Afterward, delete the respective row and column to achieve the optimal assignment.

Step 5: If in the corresponding flight there is no unique pairing (two or more flights have minimal rest time in the same pairing) then:

Identify the row that have minimal rest time within the same pairing column. Then, in that pairing column calculate the difference between the minimal and maximal rest time. Flight that has greatest difference assign it to the pairing, and then remove the corresponding column and row. If there is equalize between two or more pairings, use the difference

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between the minimal and the second highest rest time. If the second highest time isn't unoccupied, subtract the minimal with zero.

Step 6: Recalculate the minimal rest time for all flight trips. Repeat process 4 and 5 till all flight trips have been assigned uniquely to the set of pairings.

Step 7: When every flight is allocated to pairing, use this expression to calculate the whole rest time.

$$S = \sum_{l \in F} \sum_{k \in P} \Omega_{lk}\, x_{lk} \ldots (5.1)$$

6. NUMERECAL EXAMPL OF ROUND TRIP FLIGHTS AMONG COUPLE OF CITIES

The data of Pakistan International Airways (PIA) is given bellow:

Example 6.1: The airways operates the following flight schedule between Islamabad and Lahore for seven days a week.

Flight No.	Islamabad	Lahore	Flight No	Islamabad	Lahore
	Dep	Arr		Dep	Arr
011	07:00 am	08:00 am	1010	08:00 am	09:00 am
022	08:00 am	09:00 am	1020	09:00 am	10:00 am
033	01:00 pm	02:00 pm	1030	12:00 pm	01:00 pm
044	06:00 pm	07:00 pm	1040	05:00 pm	06:00 pm

Solution:

Determine the pairing for the given flights that minimizes the rest time away from the base. Each crew will be assigned to the base that results with shortest layover. The crew base for every pairing is specified. For finding optimality of the assignment, compute the rest time based on the given schedule by subtracting the departure and arrival times. Record the layover times in given Table 6.1.1.

Table 6.1.1: Displays the Layover Time at Islamabad Base

Islamabad Base				
Crew	Q	W	Y	Z
Flight No	**1010**	**1020**	**1030**	**1040**
011	24:00	01:00	04:00	09:00
022	01:00	24:00	03:00	08:00
033	18:00	19:00	22:00	03:00
044	13:00	14:00	17:00	22:00

Table 6.1.2: Displays the Layover Time at Lahore Base

Lahore Base				
Crew	Q	W	Y	Z
Flight No	**1010**	**1020**	**1030**	**1040**
011	22:00	21:00	18:00	13:00
022	23:00	22:00	19:00	14:00
033	04:00	03:00	24:00	19:00
044	09:00	08:00	05:00	24:00

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The detailed layover timetable or matrix (Table 6.1.3) is created by selecting the minimal value from the corresponding entries in Table 6.1.1 and Table 6.1.2. The minimal layover time is highlighted in red. The city with the shortest layover time will be designated as the crew base.

Table 6.1.3: Displays minimum time of layover from above tables

Crew	M	N	S	G
No of Flights	**1010**	**1020**	**1030**	**1040**
011	22:00	01:00	04:00	09:00
022	01:00	22:00	03:00	08:00
033	04:00	03:00	22:00	03:00
044	09:00	08:00	05:00	22:00

Then apply Column Penalty Method

1st **Column**	**2nd Column**
011	1020
022	1010
033	1020, 1040
044	1030

Flight **022** and **044** with single rest time at **1010**, **1030** assign these flights and delete respected columns and rows.

Crew	N	G
Flight No	**1020**	**1040**
011	01:00	09:00
033	03:00	03:00

1st Column	**2nd Column**	**Difference**
011	1020	03:00 – 01:00 = 02:00
033	1020	03:00 – 03:00 = 00:00

- Maximal deference in light **011** , assign it to **1020** and **033** to **1040**

The optimality of the assignment as below

	No. of Flight	**No. of Flight**		
Duty Crew	**ISB – LHE**	**LHE – ISB**	**layover time**	**based city**
M	011	1020	01:00	ISB
N	022	1010	01:00	LHE
S	033	1040	03:00	ISB
G	044	1030	05:00	LHE
			Total 10:00 hrs	

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7. NUMERICAL EXAMPLE OF FLIGHTS AMONG MULTIPLE CITIES

Example 7.1: Displays the daily schedule for six flight legs. Every row in the table represents a flight, detailing the number of flights, departure and arrival time and base for each leg.

Table 7.1.1: Represents flight legs among base and timing of departure and arrival

Flight Leg	DEP City	ARR City	DER Time	ARR Time
f10	KHI	ISB	2:00	4:00
f20	ISB	KHI	6:10	8:10
f30	KHI	LHE	9:20	11:21
f40	LHE	KHI	12:00	14:07
f50	ISB	LHE	15:11	17:08
f60	LHE	ISB	3:14	5:30

Solution

I – As long as Karachi is the base of the flight trip

Pairing	Rest Time
PA : {f10, f20}	*02:10 hours*
PB : {f30, f40}	*01:10 hour*
PC : {f30, f60, f20}	*f30 ⟶ f60 = 15:53, f60 ⟶ f20 = 00:40, 15:53 + 00:40 =**16:33 hrs***
PD : {f10, f50, f40}	*f10 ⟶ f50 =10:54, f50 ⟶ f40 = 18:52, 10:54 + 18:52 = **29:46 hrs***
PE : {f10, f50, f60, f20}	*f10 ⟶ f50 = 10:54, f50 ⟶ f60 = 10:06, f60 ⟶ f20 = 00:40 10:54 + 10:06 + 00:40 = **21:40 hrs***
PF : {f30, f60, f50, f40}	*f30 ⟶ f60 = 03:53, f60 ⟶ f50 = 09:41, f50 ⟶ f40 = 18:52 03:53 + 09:41+ 18:52 = **32:37 hrs***

Table 7.1.2: Represents the rest time between flights at Karachi

Crew	CA	CB	CC	CD	CE	CF
	PA	**PB**	**PC**	**PD**	**PE**	**PF**
f10	02:10	-	-	29:46	21:40	-
f20	02:10	-	16:33	-	21:40	-
f30	-	01:10	16:33	-	-	32:37
f40	-	01:10	-	29:46	-	32:37
f50	-	-	-	29:46	21:40	32:37
f60	-	-	16:33	-	21:40	32:37

II - - If ISB is the base of the flight

Pairing	Rest Time
PA : {f20, f10}	***17:50 hrs***
PB : {f20, f30, f60}	*f20 ⟶ f30 = 01:10, f30 ⟶ f60 = 15:53, 01:10 +17:03 = **17:03 hrs***
PC : {f50, f60}	***10:06 hrs***
PD : {f50, f40, f10}	*f50 ⟶ f40 = 18:52, f40 ⟶ f10 = 10:53, 18:52 + 10:53 = **29:46 hrs***
PE : {f50, f40, f30, f60}	*f50 ⟶ f40 = 18:52 , f40 ⟶ f30 = 19:17, f30 ⟶ f60 = 15:53 18:52 + 19:17 + 15:53 = **54:02 hrs***
PF : {f20, f30, f40, f10}	*f20 ⟶ f30 = 01:10, f30 ⟶ f40 = 00:39, f40 ⟶ f10 = 10:53 01:10 + 00:39+ 10:53 = **12:42 hrs***

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Table 7.1.3: Represents the layover time between flights at Islamabad

Crew	CA	CB	CC	CD	CE	CF
	PA	PB	PC	PD	PE	PF
f10	17:50	-	-	29:46	-	12:42
f20	17:50	17:03	-	-	-	12:42
f30	-	17:03	-	-	54:02	12:42
f40	-	-	-	29:46	54:02	12:42
f50	-	-	10:06	29:46	54:02	-
f60	-	-	10:06	-	54:02	-

III - If LHE is the base of the flight

Pairing	Rest Time
PA : {f40, f30}	***19:17 hrs***
PB : {f40, f10, f50}	*f10 ⟶ f40 = 10:53 , f10 ⟶ f50 = 10:54,* ***21:47 hrs***
PC : {f60, f50}	***09:41 hrs***
PD : {f60, f20, f30}	*f60 ⟶ f20 = 00:40, f20 ⟶ f30 = 01:10,* ***01:50 hrs***
PE : {f40, f10, f20, f30}	*f40 ⟶.f10 = 10:53, f10 ⟶ f20 = 02:10, f20 ⟶ f30 = 01:10,* ***14:13 hrs***
PF : {f60, f20, f10, f50}	*f60 ⟶ f20 = 00:40 , f20 ⟶ f10 = 17:40, f10 ⟶ f50 = 10:54,* ***29:31 hrs***

Table 7.1.4: Represents the rest time between flights in at LHE

Crew	CA	CB	CC	CD	CE	CF
	PA	PB	PC	PD	PE	PF
f10	-	21:47	-	-	14:13	29:31
f20	-	-	-	01:50	14:13	29:31
f30	19:17	-	-	01:50	14:13	-
f40	19:17	21:47	-	-	14:13	-
f50	-	21:47	09:41	-	-	29:31
f60	-	-	09:41	01:50	-	29:31

Table 7.1.5: Represents the minimum rest time from previous tables

Crew	CA	CB	CC	CD	CE	CF
	PA	PB	PC	PD	PE	PF
f10	02:10	21:47	-	29:46	14:13	12:42
f20	02:10	17:03	16:33	01:50	14:13	12:42
f30	19:17	01:10	16:33	01:50	14:13	12:42
f40	19:17	01:10	-	29:46	14:13	12:42
f50	-	21:47	09:41	29:46	-	29:31
f60	-	-	09:41	01:50	21:40	29:31

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Apply Column Penalty Method.

1st Column	2nd Column
f10	PA
f20	PD
f30	PB
f40	PB
f50	PC
f60	PD

f10 and ***f50*** have unique layover time at **PA** and **PC** directly assign those flights and delete columns and rows.

Crew	CB	CD	CE	CF
	PB	**PD**	**PE**	**PF**
f20	17:03	01:50	14:13	12:42
f30	01:10	01:50	14:13	12:42
f40	01:10	29:46	14:13	12:42
F60	-	01:50	21:40	29:31

1st Column	2nd Column	Difference
f20	PD	29:46 – 01:50 = 28:04
f60	PD	01:50 – 01:50 = 00:00

Allocate ***f20*** to **PD** and ***f60*** to **PE, the**n delete those columns and rows.

Crew	CB	CF
	PB	**PF**
f30	01:10	12:42
f40	01:10	12:42

1st Column	2nd Column	Difference
f30	PB	01:10 – 01:10 = 00:00
f40	PB	01:10 – 00:00 = 01:10

Allocate ***f40*** to **PB** and ***f30*** to **PF**

Table 7.1.6: Represents the optimal assignment solution

Flight Legs	Pairing	Crew Duty	Base	Rest Time
f10	*PA : { f10, f20}*	CA	KHI	02:10 hrs
f20	*PD :{f60,f20,f30}*	CD	LHE	01:50 hrs
f30	*PF:{f20, f30, f40,f10}*	CF	ISB	12:42 hrs
f40	*PB :{f30,f40}*	CB	KHI	01:10 hrs
f50	*PC : {f60, f50}*	CC	LHE	09:41 hrs
f60	*PE : {f10, f50,f60, f20}*	CE	KHI	21:40 hrs

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Table 7.1.7: Presents the result of the developed method and company method in one day of the week

Pakistan International Airlines System		**Developed Method**	
Flight legs	**Rest Time**	**Flight legs**	**Rest Time**
Karachi Base			
f10+f20+f30 *KHI-ISB-KHI-LHE*	3:10 hrs + **16:00** hrs **OR** **(Overnight)**	*f10+f20* *KHI-ISB-KHI*	**2:10** hrs
f60+f50+f40 *LHE-ISB-LHE-KHI*	3:10 hrs	*f30+f40* *KHI-LHE-KHI*	**1:10** hrs
Lahore Base			
		f60+f50 *LHE-ISB- LHE*	**9:41** hrs
Total hrs / Day	**20:10** hrs		**13:00** hrs
Salaries + Flying Time / Day	(2,960 $) / Day		(2,220 $) / Day
Overnights / Week	2,800 $		**-**

8. RESULTS AND DISCUSSION

This article focuses on the Air Crew Assignment Problem (ACAP), which involves assigning the required flights to a set of pairings (each with crew members), with a purpose to minimize the time away from the base and the cost as well, as it was clear in table (7.1.7) where the company cost was (2,960 $) / Day, and CPM method reduced that cost to (2,220 $) / Day. Connection constraints are the main constraints, ensuring that one pairing at least covers every flight and every pairing is covered by single crew. Various approaches have been taken by existing methods regarding this problem. The Hungarian method (Kuhn, 1995), for instance, is used for direct round trips among couple of cities, while other methods such as (Anbil, 1991), (Gopalakrishnan, 2005) and (Radu, 2018) focus on round trips implicating connections among several cities.

a. Case of Flights among couple of Cities

The optimal solution found by Hungarian method (HM) and Column Penalty Method (CPM) in direct flight case among couple of cities is given in Table 8.1.1. These cases are taken from Pakistan International Airways (PIA).

Table 8.1.1: Displays the difference among the solution of Hungarian method (HM) and Column Penalty Method (CPM)

Case No.	**Matrix order**	**No. of platforms within solution**		**Waiting time between flights**
		CPM	**HM**	
Example 6.1	4 X 4	1	6	10 : 00 hrs
Example 7.1	6 X 6	2	5	15 : 10 hrs
Example 3	5 X 5	3	6	15 : 00 hrs
Example 4	4 X 4	2	5	12 : 00 hrs

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The results obtained using the Column Penalty Method (CPM) (R. M. Ahmed, 2020) and the Hungarian Method (HM) (Kuhn, H.W, 1991) for direct flight legs between two cities, as illustrated in Example 6.1, are identical (10:00 hours). However, the number of tables and iterations required for each method differs. The CPM uses fewer tables, making it a faster and more efficient method as compared to the Hungarian Method.

b. The Case of Flights among Multiple Cities

The CPM was also applied to the entire week's flight schedule for real case Example 7.1, resulting in a total of 42 generated pairings. By selecting the best subsets from these pairings, the CPM method achieved an objective function value of 91 hours per week in rest time, with crew members returning to the base in each pairing after a few iterations.

For comparison, the airlines's existing techniques for the entire week of crew pairings resulted in 133 hrs of rest time and overnights stays away from the city base, in addition to the rest time within every pairing. The value was the sum of results for every day of the week. Every night away from the city base incurs an expenses or cost of $100 for each pilot and $50 for each cabin crew member.

Table 8.2.1: shows the difference between the best subsets of developed method and company method in one day of the week

Developed Method				Company Method			Cost/ Day
Flight Pairings	Base City	Rest time	Cost / Day	Flight Pairings	Base City	Rest Time	
PA:{f10,f20}	KHI	02:10 hrs	2,220 $	*f10+f20+f30* *f60+f50+f40*	KHI	3:10 hrs + 16:00 hrs OR Overnight	2,960 $
PB:{f30,f40}	KHI	01:10 hrs					
PC:{f50, f60}	LHE	09:41 hrs					

In both the developed method and the company's approach, the costs of flight pairings are the same. However, the company incurs higher charges for overnights, which increases the total time away from base. The column representing pairings shows series of flight legs that begins and ends at the same city. For example, PA includes Flight 10 from Karachi to Islamabad and Flight 20 from Islamabad to Karachi, PB includes Flight 30 from Karachi to Lahore and Flight 40 from Lahore to Karachi, and PC includes Flight 50 from Islamabad to Lahore and Flight 60 from Lahore to Islamabad.

The "Rest Time" column indicates the time among flights within every pairing, while the "Cost/Day" column present cost of associated with every pairing. Every pairing requires a crew of six members: two as cockpit and four as cabin crew. The cockpit crew members receive a daily salary of Rs.10,000/-, an additional RS.5,000/- for every hour on air, and $100 for overnight compensation. The cabin crew members receive a daily salary of Rs.2,000/-, Rs.2,000/- for every hour on air, and $50 for overnight compensation.

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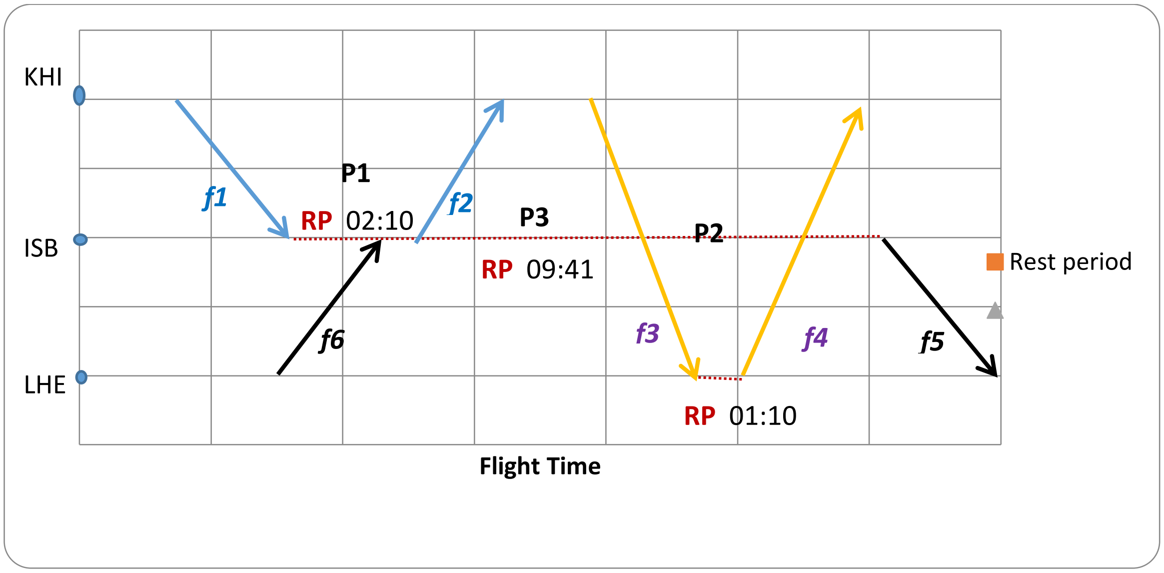

Figure 8.2.2: Presents the optimality of pairing with minimal rest time away from city base

When evaluating the CPM method against the company's current approach for each day of a specific week, the developed method demonstrates superior performance. Additionally, when compared with other established methods, such as traditional approaches (Gopalakrishnan, 2005) and (Andersson, 1998), which generate all possible pairings as variables for the optimization problem but fail to produce pairings that return to the base or specify which crew will fly these pairings, the CPM method stands out. It offers more feasible pairings with multiple bases and reduced rest periods.

In contrast, the Round-Trip solution used in the Robust Airline Crew Pairing Optimization for Short-Haul Flights, which employs a mixed-integer programming solver from Google OR (Radu, 2018), relies on traditional techniques such as Branch and Bound (Lawler, 1966), Branch and Price, and Partial Pairing Generation (Ranga, 1991) whereas the Column Penalty Method (CPM) has been used in this research for solving the assignment (optimization) problem.

9. CONCLUSION

Airline Crew Assignment Problem (ACAP) is a critical component of airline operational planning. This research article adopts an integrated modeling methodology for ACAP, aimed at assigning each crew to a base with minimal rest periods to reduce overall costs and time away from base. The concept of pairings is intended to minimize crew scheduling costs, but since the process is divided into crew pairing and crew assignment, the final solution tends to be heuristic. Therefore, integrating these two stages lead to an optimal solution.

Although the CPM model is effective, it may sometimes generate a high number of pairings, which can increase optimization time. A potential extension of this research would be to preprocess matrix constraints to reduce the number of iterations, matrix size, and consequently, the optimization processing time.

The traditional crew pairing optimization framework involves generating all feasible pairings and then optimizing them. In contrast, the presented framework consists of three stages: first, generating all feasible pairings that start and end at the same base to cover the entire planning period and meet all constraints; second, optimizing these pairings by selecting those with the minimum rest period; and third, further optimizing using the Column Penalty Method.

Existing methods do not apply universally to all types of crew assignment problems, while CPM is versatile and can be used across different airlines and scheduling scenarios. Other approaches typically focus on reducing flying time, whereas the presented method emphasizes minimizing time away from the base. Most researchers have used Google mixed-integer programming solutions, the Branch-and-Bound solver, etc, whereas this research uses the Column Penalty Method.

This method offers significant benefits to airlines, as it can be applied throughout the planning process, from fleet assignment to daily operations. It helps create more efficient flight and crew schedules, enhancing overall operational efficiency.

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