

DECISION MAKING AND OPTIMIZATION OF WATER TREATMENT PLANTS BASED ON M-POLAR FUZZY SOFT SET

KHAYRI ABU ISBAYHAH

Mathematics Department, Al Zintan University, Libya.

FATIMAH.A.R MASOUD

Mathematics Department, Al Zintan University, Libya.

ZEINEB K.A. ASHANDOULI

Mathematics Department, Bani Waleed University, Libya.

ZAYNAB. A. MAKARI

Mathematics Department, Gharyan University, Libya.

MOHAMED ABULGASIM ABUAJIYLA

Mathematics Department, Gharyan University, Libya.

ALAMIN ABUSBAIHA*

Mathematics Department, Gharyan University, Libya.

*Corresponding Author Email: alamenbb11@gmail.com

MOHAMMED M. KHALAF

Department of Industrial Engineering, Faculty of Engineering and Computer Science, Mustaqbal University, Buraydh, Qassim, Saudi Arabia.

Abstract

This study aims to identify the most suitable water treatment plant among various alternatives, including, surface water, groundwater, RO seawater desalination, thermal desalination (MSF/MED), secondary and tertiary wastewater treatment, industrial, stormwater, and sludge or advanced reuse treatment plants. The evaluation considers key technical, economic, and environmental criteria such as water quality, capital and maintenance costs, energy consumption, environmental impact, waste generation, implementation time, operational efficiency, operational risk, and space requirements. The M-polar fuzzy soft set method is applied to systematically handle uncertainty and select the optimal water treatment plant.

Keywords: Surface Water Treatment Plants, Operational Efficiency, Optimization, Desalination.

AMS Classification: 03E72, 47S40.

1. INTRODUCTION

All water treatment plants are designed with the same goal: to provide clean and safe water as efficiently and reliably as possible. However, as technology has evolved, the types and methods of water treatment have also advanced. The introduction of innovative and sustainable treatment methods has led to the development of various types of water treatment plants. Surface water treatment plants treat water from rivers, lakes, or reservoirs. They typically involve processes such as sedimentation, filtration, and disinfection. These plants can be classified into conventional treatment plants, direct

filtration plants, and slow sand filtration plants depending on the treatment process used. Groundwater treatment plants treat water extracted from wells or aquifers. These plants often focus on removing minerals, iron, manganese, and other dissolved substances. Treatment methods may include aeration, filtration, and chemical dosing depending on the water quality. RO seawater desalination plants derive freshwater from seawater using reverse osmosis technology. Water is forced through semipermeable membranes that remove salts and other impurities. This method requires significant energy but provides a reliable source of potable water in areas with limited freshwater resources. Thermal desalination plants (MSF/MED) produce freshwater by evaporating seawater and condensing the steam. They are often used in regions with high salinity water or where large-scale water production is needed. Wastewater treatment plants, both secondary and tertiary, treat sewage or industrial effluents to produce water that can be safely discharged or reused. Secondary treatment typically removes organic matter and suspended solids, while tertiary treatment further removes nutrients and pathogens to allow for safe reuse.

Industrial and stormwater treatment plants focus on treating process water or rainwater runoff. Industrial plants remove specific chemical pollutants and contaminants, whereas stormwater plants prevent pollutants from entering natural water bodies. Sludge or advanced reuse treatment plants process residual sludge from other treatment processes, recovering water and nutrients, and in some cases producing water suitable for agricultural or industrial reuse. Each type of water treatment plant is evaluated based on various characteristics such as water quality output, capital and maintenance cost, energy consumption, environmental impact, waste generation, implementation time, operational efficiency, operational risk, and space or infrastructure requirements. These factors guide the design, operation, and selection of the most suitable water treatment plant for a given context.

The remainder of this paper is organized as follows: Sections (1) and (2) present the theoretical framework of water-based power plants, along with an explanation of the concept of m -polar fuzzy sets. These sections provide a detailed presentation and analysis of the main types of hydropower-based generation systems, including hydroelectric power plants, wave power plants, and tidal range power plants. These power plants are evaluated based on a set of fundamental criteria, namely: renewable energy efficiency, visual impact, capital cost, maintenance cost, environmental impact, greenhouse gas emissions, implementation time, cost per kWh, risk of cascading accidents, and waste management issues.

Moreover, these sections introduce the concepts of the Soft Set, Fuzzy Soft Set, and Fuzzy Polar Soft Set. In Section (3), the analysis of water-based power plants and their applications is conducted using fuzzy set theory and a decision-making framework based on the 2-polar fuzzy soft set. Within the problem formulation, the proposed algorithms are presented, and the optimal selection of water-based power plants is determined according to the 2-polar fuzzy soft set decision-making criteria.

2. PRELIMINARIES

2. 1 Soft Set Theory and m-Polar Fuzzy Soft Set Model

The results of these studies are analyzed using the proposed criterion, through which a fuzzy soft set–based decision is obtained by taking into account the values of all analyses related to the characteristics of different types of water-based power plants, including hydroelectric, wave, and tidal range power plants. This approach enables an accurate comparison among these systems based on multiple technical, environmental, and economic criteria. On the other hand, Majumdar and Samanta introduced the concept of generalized fuzzy soft sets, followed by several subsequent studies on advanced models such as generalized multi-fuzzy soft sets, generalized intuitionistic fuzzy soft sets, generalized fuzzy soft expert sets, and generalized interval-valued fuzzy soft sets. Recently, Zhu and Zhan proposed the concept of fuzzy parameterized fuzzy soft sets along with their applications in decision making, while Zhao et al. presented a novel decision-making approach based on intuitionistic fuzzy soft sets. Moreover, Deli introduced the concept of interval-valued neutrosophic soft sets and its application to decision making. Furthermore, Fatimah et al. extended several models, including N-soft sets, as well as hybrid models such as interval-valued fuzzy soft sets and (dual) probabilistic soft sets. In view of these developments, this study highlights the concept of the possibility m-polar fuzzy soft set as a novel and powerful model for analyzing and optimally selecting water-based power plants based on their key characteristics, such as energy efficiency, environmental impact, economic cost, safety, and sustainability.

The results of these analyses are evaluated using the proposed criterion, through which a fuzzy soft set–based decision is obtained by considering the values of all evaluation results associated with the different water-based power plant alternatives. These alternatives include hydroelectric, wave, and tidal range power plants, which are assessed through multiple criteria related to their technical performance, environmental impact, economic cost, safety, and sustainability. This framework allows a comprehensive and systematic comparison among the various types of water-based power generation systems. On the theoretical side, Majumdar and Samanta [6] introduced the concept of generalized fuzzy soft sets, which was followed by extensive studies on advanced extensions such as generalized multi-fuzzy soft sets [7], generalized intuitionistic fuzzy soft sets [8, 9], generalized fuzzy soft expert sets [10], and generalized interval-valued fuzzy soft sets [11]. More recently, Zhu and Zhan [12] proposed the concept of fuzzy-parameterized fuzzy soft sets with applications in decision-making problems, while Zhao et al. [13] introduced a novel decision-making approach based on intuitionistic fuzzy soft sets. Furthermore, Deli [14] presented the concept of interval-valued neutrosophic soft sets and demonstrated its applicability in decision making. In addition, Fatimah et al. [15, 16] extended the existing models by introducing N-soft sets, as well as hybrid models such as interval-valued fuzzy soft sets and (dual) probabilistic soft sets. In view of these developments, this study highlights the concept of the possibility m-polar fuzzy soft set as a new and effective decision-making model that is well suited for the evaluation and

optimal selection of water-based power plants based on their key characteristics, including energy efficiency, environmental sustainability, capital and maintenance costs, implementation time, safety risks, and waste management issues. Let E be a non-empty finite set of attributes (parameters, characteristics or properties) which the objects in U possess and let $P(U)$ denote the family of all subsets of U . Then a soft set is defined with the help of a set-valued mapping as given below:

Definition 2.1.1 (Molodtsov [6]) A pair (F, A) is called a soft set over U , where $A \subseteq E$ and $F: A \rightarrow P(U)$ is a set-valued mapping. In other words, a soft set (F, A) over U is a parameterized family of subsets of U where each parameter $e \in A$ is associated with a subset $F(e)$ of U . The set $F(i)$ contains the objects of U having the property i and is called the set of i -approximate elements in (F, A) .

Definition 2.1.2 (Chen, Li and Koczy, [17,18]) Elements $([0,1]^m)^X$ the set of all mappings from X to $[0,1]^m$ with the point – wise order are called an m -polar fuzzy sets, such that m is an arbitrary cardinality. A subset $\mathcal{A} = \{\mathcal{A}_k\}_{k \in K} \subseteq ([0,1]^m)^X$ (or a mapping $\mathcal{A}: K \rightarrow ([0,1]^m)^X$ satisfying $\mathcal{A}(k) = \mathcal{A}_k \forall k \in K$) is called an an m -polar fuzzy soft set on X .

Example 2.1.1 Let $X = \{a_1, a_2\}$ be a two element set, $I = \{i_1, i_2, i_3\}$ be a four element set, the 2-polar fuzzy soft set $\mathcal{A} \in [([0,1]^2)^X \times ([0,1]^2)^X]^I$ defined by:

$$\mathcal{A}(a_1) = \left\{ \frac{(0.6,0.4)}{i_1}, \frac{(0.24,0.5)}{i_2}, \frac{(0.9,0.3)}{i_3} \right\}$$

$$\mathcal{A}(a_2) = \left\{ \frac{(0.90,0.987)}{i_1}, \frac{(0.654,0.123)}{i_2}, \frac{(0.897,0.879)}{i_3} \right\}$$

Definition 2.3 Let $\{\mathcal{A}_k\}_{k \in K} \in [([0,1]^m)^X]^I$. Define m -polar fuzzy soft sets

$$\bigvee \{\mathcal{A}_k\}_{k \in K} = \max \{\mathcal{A}_k\}_{k \in K} \quad \text{and} \quad \bigwedge \{\mathcal{A}_k\}_{k \in K} = \min \{\mathcal{A}_k\}_{k \in K} .$$

3. ANALYSES OF WATER STATIONS

In the next table explains the Analyses of power plants and the degree of all to state the optimal power plant, this degree is reality degree. Table 1, figures 1 and 2. Explains Analyses of power plants. (In table 1, figure 1, table 2 and figure 2. We analysis a power plants from reality values of the stations)

Table (1): Comparison Approach for Water Treatment Plants (Main Properties)

Water Treatment Plant Type	Water Source Suitability	Capital Cost	Maintenance Cost	Energy Consumption	Environmental Impact
Surface Water Treatment Plants (SWTP)	0.8	0.6	0.5	0.4	0.5
Groundwater Treatment Plants (GTP)	0.7	0.4	0.3	0.3	0.3

Seawater Desalination (RO) (SD -RO)	1.0	0.9	0.7	0.9	0.8
Thermal Desalination (MSF/MED) (TD - MSF/MED)	1.0	1.0	0.8	1.0	0.9
Wastewater Treatment – Secondary (WTS)	0.6	0.5	0.5	0.4	0.4
Tertiary Treatment Plants (TTP)	0.9	0.7	0.6	0.5	0.3
Industrial Water Treatment Plants (IWTP)	0.7	0.8	0.7	0.7	0.7
Stormwater Treatment Plants (STP)	0.5	0.3	0.2	0.2	0.2
Sludge / Advanced Reuse Treatment Plant (S RTP)	0.9	35	13.0	0.7	0.9

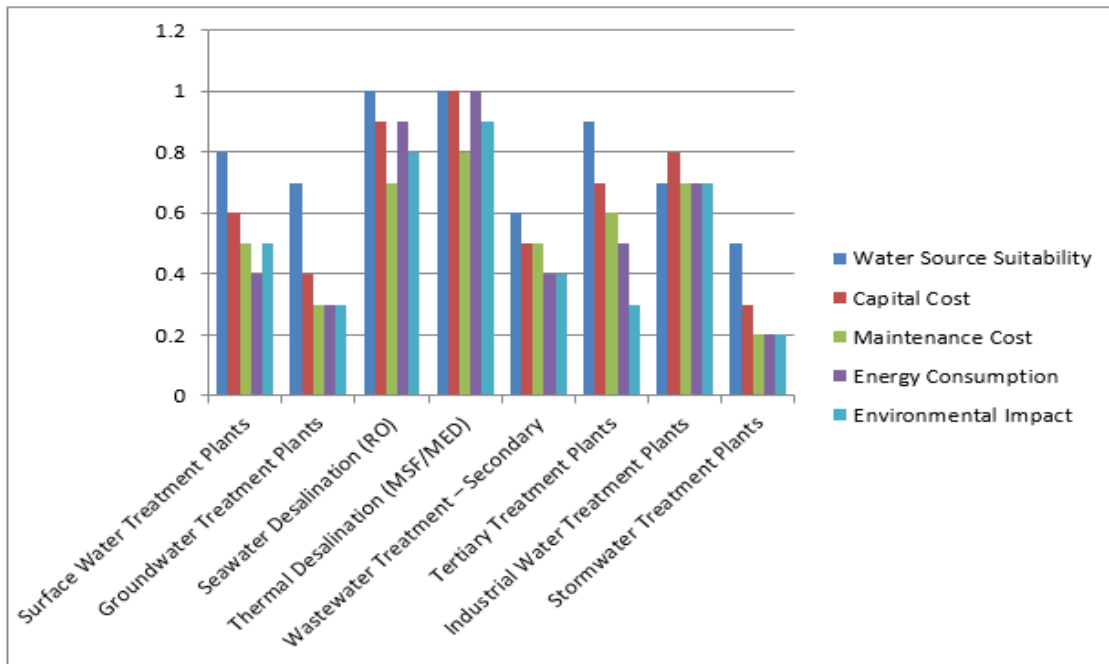


Figure 1: Analyses of Water stations

Table (2): Comparison Approach for Water Treatment Plants (Operational Properties)

Water Treatment Plant Type	Treatment Efficiency	Implementation Time	Cost (Cent/m ³)	Operational Risk	Waste Generation
Surface Water Treatment Plants (SWTP)	0.5	18	6.5	0.4	0.4
Groundwater Treatment Plants (GTP)	0.4	15	5.5	0.3	0.3
Seawater Desalination (RO) (SD -RO)	0.9	30	12.5	0.6	0.7
Thermal Desalination (MSF/MED) (TD - MSF/MED)	1.0	45	15.0	0.7	0.8

Wastewater Treatment – Secondary (WTS)	0.6	20	7.0	0.4	0.5
Tertiary Treatment Plants (TTP)	0.7	25	9.0	0.5	0.5
Industrial Water Treatment Plants (IWTP)	0.8	28	10.5	0.6	0.7
Stormwater Treatment Plants (STP)	0.3	12	4.5	0.3	0.3
Sludge / Advanced Reuse Treatment Plant (SRTP)	0.9	35	13.0	0.7	0.9

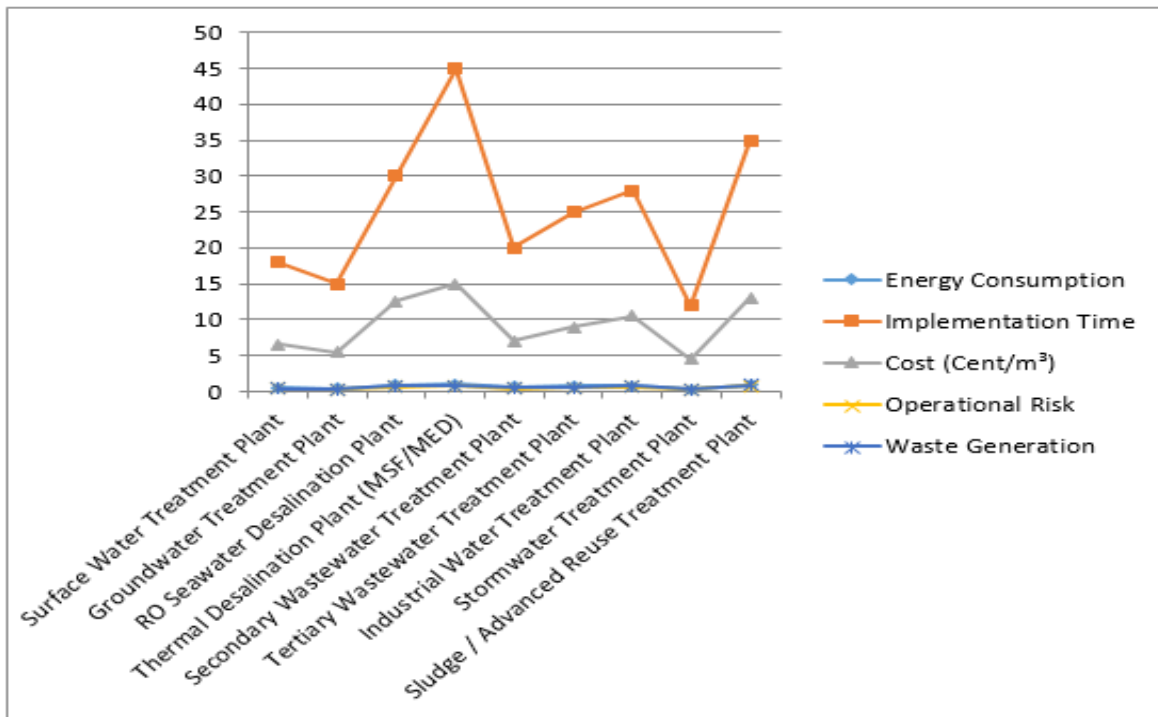


Figure 2: Analyses of Water stations

3.1. Optimal alternative for Water stations by *m*-polar fuzzy soft set

In this study, we selected a comprehensive set of water treatment plants, including surface water treatment plants, groundwater treatment plants, RO seawater desalination plants, thermal desalination plants (MSF/MED), secondary wastewater treatment plants, tertiary wastewater treatment plants, industrial water treatment plants, stormwater treatment plants, and sludge or advanced reuse treatment plants.

The main objective of this study is to determine the optimal alternative for the suitability and efficient operation of water treatment plants under a variety of evaluation criteria. These criteria include water quality, capital cost, maintenance cost, energy consumption, environmental impact, waste generation, implementation time, operational efficiency, operational risk, and space or infrastructure requirements.

To properly handle the uncertainty and imprecision associated with evaluating multiple alternatives, the M-polar fuzzy soft set method is employed. This approach allows a systematic and rigorous analysis of the performance of all selected water treatment plants according to the aforementioned criteria.

By applying this methodology, all types of water treatment plants—surface water, groundwater, RO seawater desalination, thermal desalination (MSF/MED), secondary and tertiary wastewater treatment, industrial water treatment, stormwater treatment, and sludge or advanced reuse treatment plants—are evaluated comprehensively across the ten criteria, enabling the identification of the most suitable and effective water treatment plant for a given context.

Let the set $X = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8, \alpha_9\}$ express about the Water stations (Surface water treatment plant, Groundwater treatment plant, RO Seawater desalination plant, Thermal desalination plant (MSF/MED), Secondary wastewater treatment plant, Tertiary wastewater treatment plant, Industrial water treatment plant, Stormwater treatment plant, Sludge or advanced reuse treatment plant) with the parameters $I = \{\rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6, \rho_7, \rho_8, \rho_9, \rho_{10}\}$

where ρ_1 stands for Water Source Suitability, ρ_2 stands for Capital Cost, ρ_3 stands for Maintenance Cost, ρ_4 stands for Energy Consumption, ρ_5 stands for Environmental Impact, ρ_6 stands for Energy Consumption, ρ_7 stands for Implementation Time, ρ_8 stands for Cost (Cent/m³), ρ_9 stands for Operational Risk, and ρ_{10} stands for Waste Generation. these parameters are important with degree.

We will now formulate an algorithm to solve the decision-making problem as described below

Algorithm 1: Decision-Making Using 3-Polar Fuzzy Soft Set

Step 1. State $\mathcal{A} \in ([0,1050]^3)^X$.

Step 2. Compute $p_k \circ \bar{\mathcal{A}} = 1050 \wedge \sum_{i \in I} p_k \circ \mathcal{A}(x)$ ($\forall x \in X$), where $p_k: [0,1050]^3 \rightarrow [0,1050]$ is the k -the projection ($k = 1, 2, 3$).

Step 3. Compute $\bar{\mathcal{A}} \in ([0,1050]^3)^X$.

Step 4. Put a suitable weigh vector $e^{\rightarrow} = (1.00, -10.00, -100.00)^T$. And compute the score $S(x) = \bar{\mathcal{A}}(x)e^{\rightarrow}$ for each $x \in X$.

Step 5. The maximal value of $S(\tilde{x})$ state the optimal alternative for suitability of power plants based on 3-polar Fuzzy soft set.

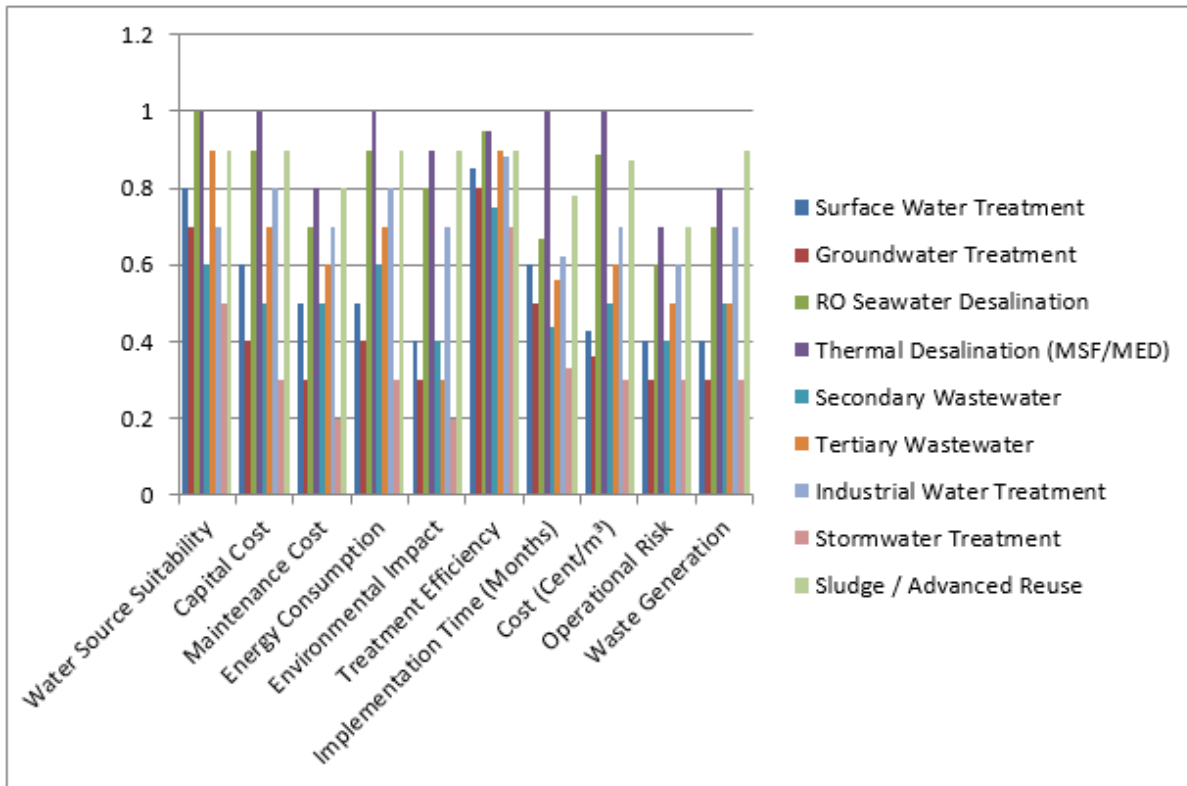
Table 3: Key values and criteria for different types of water treatment plants, including water source suitability, energy consumption, capital cost, maintenance cost, environmental impact, implementation time, cost (Cent/m³), operational risk, and waste generation

(Table 3, Follow Table 3 and Figure 3 illustrate and explain the main values and criteria for different types of water treatment plants with respect to water source suitability, energy consumption, capital cost, maintenance cost, environmental impact, implementation time, cost (Cent/m³), operational risk, and waste generation.)

Water Treatment Plant Type	Water Source Suitability	Capital Cost	Maintenance Cost	Energy Consumption	Environmental Impact
Surface Water Treatment Plants (SWTP)	0.80	0.60	0.50	0.50	0.40
Groundwater Treatment Plants (GTP)	0.70	0.40	0.30	0.40	0.30
Seawater Desalination (RO) (SD –RO)	1.00	0.90	0.70	0.90	0.80
Thermal Desalination (MSF/MED) (TD - MSF/MED)	1.00	1.00	0.80	1.00	0.90
Wastewater Treatment – Secondary (WTS)	0.60	0.50	0.50	0.60	0.40
Tertiary Treatment Plants (TTP)	0.90	0.70	0.60	0.70	0.30
Industrial Water Treatment Plants (IWTP)	0.70	0.80	0.70	0.80	0.70
Stormwater Treatment Plants (STP)	0.50	0.30	0.20	0.30	0.20
Sludge / Advanced Reuse Treatment Plant (SRTP)	0.90	0.90	0.80	0.90	0.90

Water Treatment Plant Type	Treatment Efficiency	Implementation Time (Months)	Cost (Cent/m ³)	Operational Risk	Waste Generation
Surface Water Treatment Plants (SWTP)	0.85	0.60	0.43	0.40	0.40
Groundwater Treatment Plants (GTP)	0.80	0.50	0.36	0.30	0.30
Seawater Desalination (RO) (SD –RO)	0.95	0.67	0.89	0.60	0.70
Thermal Desalination (MSF/MED) (TD - MSF/MED)	0.95	1.00	1.00	0.70	0.80
Wastewater Treatment – Secondary (WTS)	0.75	0.44	0.50	0.40	0.50

Tertiary Treatment Plants (TTP)	0.90	0.56	0.60	0.50	0.50
Industrial Water Treatment Plants (IWTP)	0.88	0.62	0.70	0.60	0.70
Stormwater Treatment Plants (STP)	0.70	0.33	0.30	0.30	0.30
Sludge / Advanced Reuse Treatment Plant (SRTF)	0.90	0.78	0.87	0.70	0.90



(Table 4 and Figure 4 illustrate and explain the main values and criteria for water treatment plants with respect to water source suitability, treatment efficiency, capital cost, maintenance cost, environmental impact, implementation time, cost (Cent/m³), operational risk, and waste generation.)

Water Source Suitability	Capital Cost	Maintenance Cost	Energy Consumption	Environmental Impact	Treatment Efficiency	Implementation Time (Months)	Cost (Cent/m ³)	Operational Risk	Waste Generation
7.1	6.1	5.1	6.1	4.9	7.68	5.5	5.65	4.5	5.1

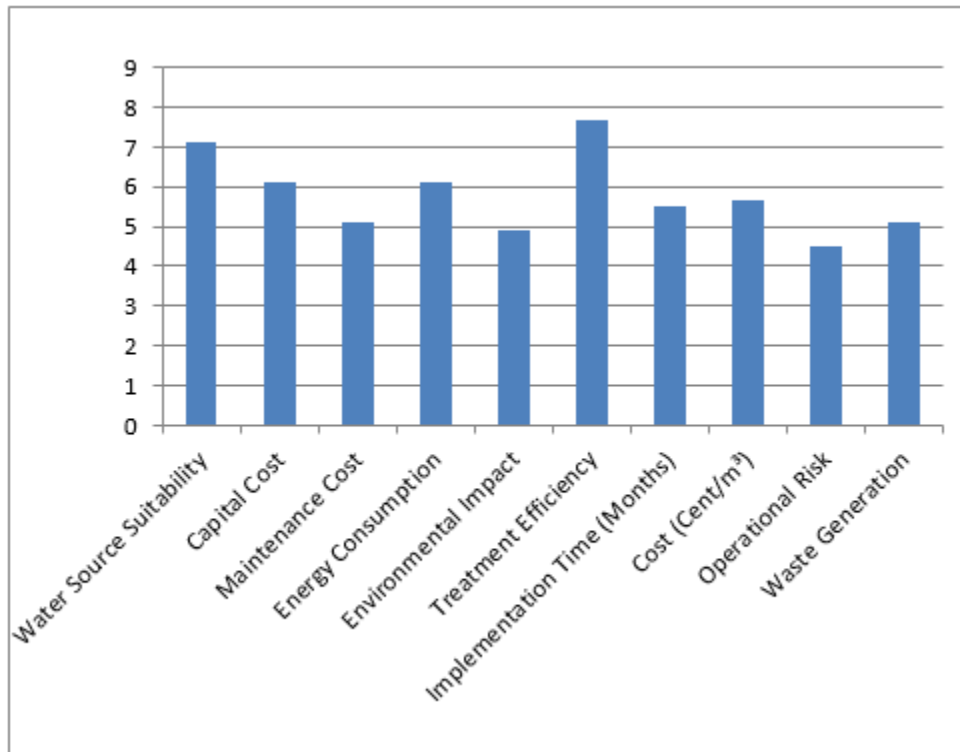


Figure 4: Illustrate and explain the main values and criteria for water treatment plants with respect to water source suitability, treatment efficiency, capital cost, maintenance cost, environmental impact, implementation time, cost (Cent/m³), operational risk, and waste generation

Tree of experts in water plants gives the degree of equality of the alternative $I = \{i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8, i_9, i_{10}\}$ for the water plants. And the data provided by the committee for decision-making use is the following 3-polar fuzzy soft set $\mathcal{A} \in [([0,100]^3)^I]^X = [([0,100]^3)^X]^I = ([0,100]^3)^{I \times X} = ([0,100]^3)^{X \times I}$ defined by:

Table 5: Triangular Values of Water Treatment stations (i_1 – i_5)

WS	i_1	i_2	i_3	i_4	i_5
x_1	(7.0, 7.1, 7.2)	(6.0, 6.1, 6.2)	(5.0, 5.1, 5.2)	(6.0, 6.1, 6.2)	(4.8, 4.9, 5.0)
x_2	(6.7, 6.8, 6.9)	(5.8, 5.9, 6.0)	(5.3, 5.4, 5.5)	(5.7, 5.8, 5.9)	(4.9, 5.0, 5.1)
x_3	(7.3, 7.5, 7.6)	(6.3, 6.5, 6.6)	(5.5, 5.7, 5.8)	(6.1, 6.3, 6.4)	(5.0, 5.2, 5.3)
x_4	(6.8, 6.9, 7.0)	(5.9, 6.0, 6.1)	(5.3, 5.5, 5.6)	(5.9, 6.0, 6.1)	(4.7, 4.8, 4.9)
x_5	(7.2, 7.3, 7.4)	(6.1, 6.2, 6.3)	(5.5, 5.6, 5.7)	(6.0, 6.2, 6.3)	(5.0, 5.1, 5.2)
x_6	(6.9, 7.0, 7.1)	(5.7, 5.8, 5.9)	(5.2, 5.3, 5.4)	(5.8, 5.9, 6.0)	(4.6, 4.7, 4.8)
x_7	(7.4, 7.4, 7.5)	(6.3, 6.4, 6.5)	(5.7, 5.8, 5.9)	(6.3, 6.4, 6.5)	(5.2, 5.3, 5.4)
x_8	(6.6, 6.7, 6.8)	(5.6, 5.7, 5.8)	(5.1, 5.2, 5.3)	(5.5, 5.7, 5.8)	(4.5, 4.6, 4.7)
x_9	(7.5, 7.6, 7.7)	(6.5, 6.6, 6.7)	(5.8, 5.9, 6.0)	(6.4, 6.5, 6.6)	(5.3, 5.4, 5.5)

Table 6: Triangular Values of Water Treatment stations (i_6-i_{10})

WS	i_6	i_7	i_8	i_9	i_{10}
x_1	(7.6, 7.68, 7.7)	(5.4, 5.5, 5.6)	(5.6, 5.65, 5.7)	(4.4, 4.5, 4.6)	(5.0, 5.1, 5.2)
x_2	(7.1, 7.2, 7.3)	(5.7, 5.8, 5.9)	(5.2, 5.3, 5.4)	(4.6, 4.7, 4.8)	(5.2, 5.3, 5.4)
x_3	(7.8, 8.0, 8.2)	(5.4, 5.6, 5.7)	(5.6, 5.8, 5.9)	(4.7, 4.9, 5.0)	(4.9, 5.0, 5.1)
x_4	(7.3, 7.5, 7.6)	(5.6, 5.7, 5.8)	(5.5, 5.6, 5.7)	(4.5, 4.6, 4.7)	(5.1, 5.2, 5.3)
x_5	(7.6, 7.8, 7.9)	(5.8, 5.9, 6.0)	(5.6, 5.7, 5.8)	(4.7, 4.8, 4.9)	(5.3, 5.4, 5.5)
x_6	(7.2, 7.3, 7.4)	(5.5, 5.6, 5.7)	(5.3, 5.4, 5.5)	(4.4, 4.5, 4.6)	(5.0, 5.1, 5.2)
x_7	(8.0, 8.1, 8.2)	(5.9, 6.0, 6.1)	(5.8, 5.9, 6.0)	(4.9, 5.0, 5.1)	(5.4, 5.5, 5.6)
x_8	(6.9, 7.0, 7.1)	(5.4, 5.5, 5.6)	(5.1, 5.2, 5.3)	(4.3, 4.4, 4.5)	(4.9, 5.0, 5.1)
x_9	(8.1, 8.2, 8.3)	(6.0, 6.1, 6.2)	(5.9, 6.0, 6.1)	(5.0, 5.1, 5.2)	(5.5, 5.6, 5.7)

Where $\mathcal{A}(x_1)(i_1) = (7.0, 7.1, 7.2)$ means that the renewable of Water station x_1 (Surface Water Treatment) is given by expert 1 (resp., by expert 2, by expert 3) is 7.0 (resp., 7.1, 7.2); that means that the power plant x_1 (Surface Water Treatment) is not renewable, the meanings of $\mathcal{A}(x_s)(i_t)$ can be explained by the same fashion ($s = 1, 2, 3, 4, 5, 6, 7, 8, 9$; $t = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$).

To find the best choice from X , let us first compute the 3-polar fuzzy set $\bar{\mathcal{A}} \in ([0, 10]^3)^X$, defined by $P_k \circ \bar{\mathcal{A}} = 10 \wedge \sum_{i \in I} p_k \circ \mathcal{A}(x)$ ($\forall x \in X$), where $p_k: [0, 100]^3 \rightarrow [0, 100]$ is the k -the projection ($k = 1, 2, 3$).

$$P_1(x_1) = 100 \wedge (7.0 + 6.7 + 7.3 + 6.8 + 7.2 + 6.9 + 7.4 + 6.6 + 7.5) = 100 \wedge 63.4 = 63.4.$$

Similarly,

Table 7: Explain the final values of compute the 3-polar fuzzy set $\bar{\mathcal{A}} \in ([0, 100]^3)^X$, defined by $p_k \circ \bar{\mathcal{A}} = 100 \wedge \sum_{i \in I} p_k \circ \mathcal{A}(x)$ ($\forall x \in X$), where $p_k: [0, 100]^3 \rightarrow [0, 100]$ is the k -the projection ($k = 1, 2$, and 3)

WS	P_1	P_2	P_3
x_1	63.4	64.3	65.2
x_2	54.2	55.2	56.1
x_3	48.4	49.5	50.4
x_4	54.7	54.9	55.8
x_5	44.0	44.0	45.9
x_6	67.6	68.78	68.7
x_7	50.7	52.7	53.6
x_8	49.6	51.55	51.4
x_9	41.5	42.5	43.4
x_{10}	46.3	46.2	48.1

Therefore,

$$\bar{A} = \left\{ \begin{array}{ccc} \frac{(63.4, 64.3, 65.2)}{x_1}, \frac{(54.2, 55.2, 56.1)}{x_2}, \frac{(48.4, 49.5, 50.4)}{x_3} \\ \frac{(54.7, 54.9, 55.8)}{x_4}, \frac{(44.0, 44.0, 45.9)}{x_5}, \frac{(67.6, 68.78, 68.7)}{x_6} \\ \frac{(50.7, 52.7, 53.6)}{x_7}, \frac{(41.5, 42.5, 43.4)}{x_8}, \frac{(46.3, 46.2, 48.1)}{x_9} \end{array} \right\}$$

Based on the weigh vector $e^{\rightarrow} = (1.00, -10.00, -100)^T$. We compute the score $S(x) = \mathcal{A}(x)e^{\rightarrow}$ for each $x \in X$. Then:

$$S(x_1) = (63.4, 64.3, 65.2) e^{\rightarrow} = 63.4 \times 1.00 + 64.3 \times (-10.0) + 65.2 \times (-100.0) = -7099.6$$

By the Model we complete to get,

WS	$S(x_i)$
x_1	-7099.6
x_2	-6107.8
x_3	-5486.6
x_4	-6074.3
x_5	-4986.0
x_6	-7490.2
x_7	-5836.3
x_8	-4723.5
x_9	-5225.7

As $S(x_8) = -4723.5$, then

(Stormwater Treatment) under the values of water treatment plants with respect to water source suitability, treatment efficiency, capital cost, maintenance cost, environmental impact, implementation time, cost (Cent/m³), operational risk, and waste generation have the highest value, the best choice by experts should be Surface Water Treatment is suitability of water stations based on m3-polar Fuzzy soft set. Depended on these results, we can rearrangement the power plant according to equality as:

Table 8: Explain the rearrangement the water stations according to equality

WS	$S(x_i)$
Stormwater Treatment	-4723.5
Secondary Wastewater	-4986.0
Sludge Advanced Reuse	-5225.7
RO Seawater Desalination)	-5486.6
Industrial Water Treatment	-5836.3
Thermal Desalination (MSF/MED)	-6074.3
Groundwater Treatment	-6107.8
Surface Water Treatment	-7099.6
Tertiary Wastewater	-7490.2

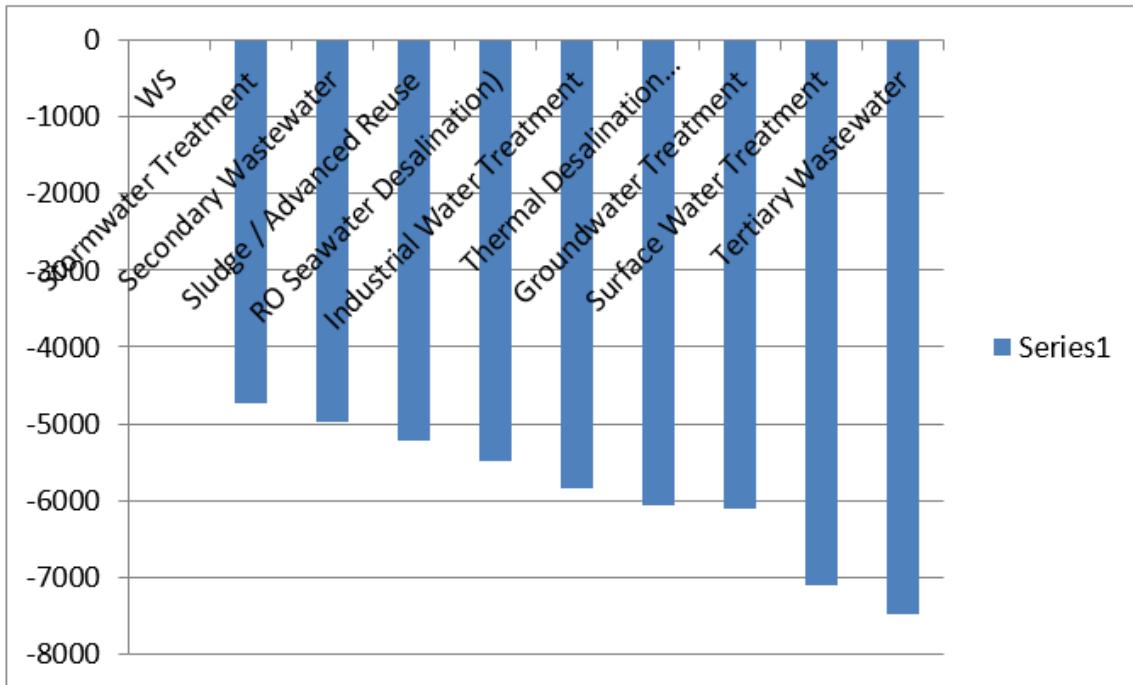


Figure 5: Explain the rearrangement the water stations according to equality

References

- 1) Weisman, Joel (1985). Modern Plant Engineering .Englewood Cliffs, New Jersey 07632: Prentice-Hall, Inc. ISBN 0-13-597252-3.
- 2) "Types of Hydropower Plants | Department of Energy". Wwww.energy. Gov. Retrieved 2018-04-18.
- 3) "What Does An Electrical Engineer Do?" Sokanu. Retrieved April 20, 2018.
- 4) Wagner, Vivian. "Engineers Who Work With Solar" Houston Chronicle. Retrieved April 20, 2018.
- 5) "What does a Civil Engineer Do?" Sokanu. Retrieved April 20, 2018.
- 6) P. Majumdar and S.K. Samanta, Generalised fuzzy soft sets, Computers and Mathematics with Applications 59 (2010), 1425–1432.
- 7) M. Pal and A. Dey, Generalised multi-fuzzy soft set and its application in decision making, Pacific Science Review A: Natural Science and Engineering 17 (2015), 23–28.
- 8) M. Agarwal, K.K. Biswas and M. Hanmandlu, Generalized intuitionistic fuzzy soft sets with applications in decision- making, Applied Soft Computing 13 (2013), 3552–3566.
- 9) A.M. Khalil, Commentary on "Generalized intuitionistic fuzzy soft sets with applications in decision-making", Applied Soft Computing 37 (2015), 519–520.
- 10) A.A. Hazaymeh, I.B. Abdullah, Z.T. Balkhi and R.I. Ibrahim, Generalized fuzzy soft expert set, Journal of Applied Mathematics 2012 (2012), 22. Article ID 328195.
- 11) S. Alkhazaleh and A.R. Salleh, Generalised interval-valued fuzzy soft set, Journal of Applied Mathematics 2012 (2012), 18. Article ID 870504.

- 12) K. Zhu and J. Zhan, Fuzzy parameterized fuzzy soft sets and decision making, *International Journal of Machine Learning and Cybernetics* 7 (2016), 1207–1212.
- 13) H. Zhao, W. Ma and B. Sun, A novel decision making approach based on intuitionistic fuzzy soft sets, *International Journal of Machine Learning and Cybernetics* 8 (2017), 1107–1117.
- 14) I. Deli, Interval-valued neutrosophic soft sets and its decision making, *International Journal of Machine Learning and Cybernetics* 8 (2017), 665–676.
- 15) F. Fatimah, D. Rosadi, R.B.F. Hakim and J.C.R. Alcantud, N-soft sets and their decision making algorithms, *Soft Computing* 22(12) (2018), 3829–3842.
- 16) F. Fatimah, D. Rosadi, R.B.F. Hakim and J.C.R. Alcantud, Probabilistic soft sets and dual probabilistic soft sets in decision-making, *Neural Computing and Applications*
- 17) J. Chen, S.-G. Li, S.-Q. Ma and X. Wang, m-Polar fuzzy sets: An extension of bipolar fuzzy sets, *The scientific world journal* **2014**: 416530
- 18) L.T. Koczy, Vectorial I-fuzzy Sets. In: M.M. Gupta and E. Sanchez, (eds.) *Approximate Reasoning in Decision Analysis*, North Holland, Amsterdam, **1982**, pp. 151-1 56.