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F_{μ}-R-RETRACT AND F_{μ}-SEMI INTERIOR MAPPING

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Abstract

In the literature, if (X, δ) is an F-ts and $Y \subset X$, the induced F-topological Vicente A novel notion of F-topological subspaces was presented in [Fuzzy Sets and Systems 58 (1993) 365], which aligns with the standard definition when $\mu = \chi_Y$. Additionally, they presented the ideas of F_{μ}-continuity and F_{μ}-open sets. In this study, we introduce weaker variants of F_{μ}-continuity using the previously mentioned ideas. Rodabough established the concept of an F-retract [J. Math. Anal. Appl. 79 (1981) 273]. The lesser versions of it are presented here. The concepts of F_{μ}-irresolute mapping, F_{μ}-semi closure, and F_{μ}-semi-interior are presented. Numerous outcomes are achieved.

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Keywords: $\mathrm{F_M PC}$, SI_{μ}, SC_{μ} and $\mathrm{F_M P}$-Retract.

INTRODUCTION

Macho Sadler and de Prada Vicente [12] cogitated the system of F-topological subspaces and F_{μ}-continuity He proved many theorems F-topological subspaces. According to Rod bough [J. Math. Anal. Appl. 79 (1981) 273], a retraction is the application of F_{μ}-continuity. Mahmoud's theory of F-topological subspaces and retractions is the best suitable theory for handling uncertainties [14]. Since then, the concept of retractions has been extensively used to various topological structures. Many writers [1,4,5,22] have examined weaker forms of F-continuity between fuzzy topological spaces utilizing the ideas of F-semi open sets [1], F-preopen sets [20], F-strongly semi open sets [2], F-semi preopen sets [7], and F-regular open sets [15]. In Section 1, we provide and examine two new F-topological concepts: F_{μ}-perfectlycontinuous, F_{μ}-completelycontinuous, and F_{μ}-R-Continuous, F_{μ}-

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perfectlyretract, We define and examine F_{μ}-completelyretract, F_{μ}-R-retract, F_{μ}-neighborhood perfectly retract, F_μ-neighborhood totally retract, and F_μ-neighborhood R-retract using these concepts in the same section. The concepts of F_{μ}-semi closure, F_{μ}-semi interior, and F_{μ}-irresolute mapping are presented in Section 2. A few of these ideas' basic characteristics are examined. For the sake of this work, let X be a nonempty set, I = [0, 1], and for every x in X, α(x) = α for every α in I. We assume that the publications [3,8,11,22,25] are well-known and refer to them for definitions and outcomes not included in this paper. See [6,10,13,14,17–19] for additional reading. Let X be a set that is not empty. A function with domain X and values in I is called a fuzzy set in X [24]. F-set and F-ts will be the abbreviations for fuzzy set and fuzzy topological space, respectively [9]. Additionally, we shall indicate the interior, closure, and complement of the F-set υ of F-topological subspace by $\mathrm{Int}_{\mu}(\upsilon), \mathrm{InCl}_{\mu}(\upsilon)$ and μ- υ, respectively.

The following definitions and findings are mentioned.

Let $(X;\ \delta)$ be an F-ts and $\mu \in I^{X}$. We call $\mathcal{A}_{\mu} = \{\upsilon \in I^{X} : \upsilon \leq \mu\}$

Definition [12]. The family $\delta_{\mu} = \{\upsilon \wedge \mu : \upsilon \in \delta\}$ is the F_{μ}-topology induced

Over μ by δ . The elements of δ_{μ} are called F_{μ}-open sets

Proposition [12]. δ_{μ} Verifying the following properties:

(i) $if\ \ \upsilon \in \delta_{\mu}$, then $\in \mathcal{A}$;

(ii) $\underline{0}$; $\mu \in \delta_{\mu}$

(iii) $if\ \ \mu_1, \mu_2 \in \delta_{\mu}$, then $\mu_1 \wedge \mu_2 \in \delta_{\mu}$;

(iv) $if\ \ \{\upsilon_j : j \in J\} \subset \delta_{\mu}\ , then\ \vee_{j \in J} \upsilon_j \in \delta_{\mu}$

Definition [12]. $\upsilon \in \mathcal{A}_{\mu}$ is a F_{μ}-closed set if $\mu - \upsilon \in \delta_{\mu}$ we note δ_{μ}^{c} the family

Of all F_{μ}-closed sets.

Lemma [1]. *For mappings* $f_i : X_i \to Y_i$ *and* F-*sets* λ_i *of* λ_i $(i = 1,2)$, *we have*

$(f_1 \times f_2)^{\leftarrow}(\lambda_1 \times \lambda_2) = f_1^{\leftarrow}(\lambda_1) \times f_2^{\leftarrow}(\lambda_2)$.

Lemma [1]. *Let* $g : X \to X \times Y$ *be the graph of a mapping* $f : X \to Y$. *Then if* λ *is a* F-*set of X and* μ *is a* F-*set of Y, then* $g^{\leftarrow}(\lambda \times \mu) = \lambda \wedge f^{\leftarrow}(\mu)$

1- F_{μ}- retracts

Definition 1.1 Let $f : (X, \delta) \to (Y, \gamma)$ be a mapping from a F- ts (X, δ) to another F- ts (Y, γ) , $\mu \in I^{X}$. Then f is called:

(i) A F- perfectlycontinuous (briefly, F_{μ}PC) mapping F_{μ} for each $\in \gamma_{f(\mu)}$,

We have $\mu \wedge f^{-1}(\upsilon)$ is both F_{μ}-open and F_{μ}-closed set of X.

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(ii) a F_μ- completelycontinuous $\left(\text{briefly, } F_\mu CC\right)$ mapping F_μ for each $v \in \gamma_{f(\mu)}$, we have $\mu \wedge f^{-1}(v)$ is regularopen set of X.

(iii) a F_μ-R-continuous $\left(\text{briefly, } F_\mu RC\right)$ mapping F_μ for each F_μ-regular open $\in \gamma_{f(\mu)}$, We have $\mu \wedge f^{-1}(v)$ is F_μ- regular open of X.

Remark 1.1 The implications between these different concepts are given

By the following diagram:

$$F_\mu PC \implies F_\mu CC \implies F_\mu RC$$

The converse of the above implication need not be true in general, as shown by the following examples.

Example 1.1 Let $X = \{a, b\}$, $Y = \{y\}$, $\delta = \{\underline{0}, \underline{1}, \lambda_1, \lambda_2, \lambda_3\}$. and

$\gamma = \{\underline{0}, \underline{1}, \theta_1, \theta_2, \theta_3\}$. $\lambda_1, \lambda_2, \lambda_3$ and $\mu \in I^X$, $\theta_1, \theta_2, \theta_3$ I^Y, defined by

$$\lambda_1 = a_{0.4} \vee b_{0.3}$$
$$\lambda_2 = a_{0.3} \vee b_{0.2}$$
$$\lambda_3 = a_{0.2} \vee b_{0.1}$$
$$\mu = a_{0.6} \vee b_{0.7}$$
$$\theta_1 = y_{0.4}$$
$$\theta_2 = y_{0.5}$$
$$\theta_3 = y_{0.6}$$

Then, the constant function *f* is F_μ- Rcontinuous, but not F_μ-C continuous.

Example 1.2 Let $X =$ Y=$\{a, b\}$, $\delta = \{\underline{0}, \underline{1}, \lambda_1, \lambda_2\}$. and

$\gamma = \{\underline{0}, \underline{1}, \theta_1, \theta_2\}$. λ_1, λ_2 and $\mu \in I^X$, θ_1, $\theta_2 \in I^Y$, defined by

$$\lambda_1 = a_{0.1} \vee b_{0.1}$$
$$\lambda_2 = a_{0.2} \vee b_{0.3}$$
$$\mu = a_{0.5} \vee b_{0.4}$$
$$\theta_1 = a_{0.3} \vee a_{0.2}$$
$$\theta_2 = a_{0.1} \vee a_{0.1}$$

$f(a) = b$, $f(b) = a$. Then f is F_μ- Ccontinuous, but not F_μ-P continuous.

Definition 1.2 $\mu \in I^X$, A F- ts (X, δ) is called a F_μ- extremally disconnected space (abbreviated as F_μED-space), μ-closure of every F_μ-open set of X is F_μ-open

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Lemma 1.1 Let (X,δ) be an F_{μ}ED- space, $\mu \in I^X$. Then, if λ is F_{μ}-regular open set of X, it is both F_{μ} –open and F_{μ} -closed

Theorem 1.1 Let (X,δ) be an F_{μ}ED- space, $\mu \in I^X$, and f:$(X,\delta) \to (Y,\gamma)$ be a mapping. Then the following are equivalent.

(i) f is F_{μ} -PC

(ii) f is F_{μ} -CC.

Proof It follows from lemma 1.1

Theorem 1.2 Let f:$(X,\delta) \longrightarrow (Y,\gamma)$ be a mapping, $\in I^X$. Then, f isF_{μ}-perfectely continuous (resp., F_{μ}- completely continuous) iff the inverse image of every F_{μ}-closed set of Y is F_{μ}-open an F_{μ}-closed (resp., F_{μ}-regular open set of X)

Proof obvious.

Theorem 1.3. Let (X,δ) , (Y,γ) be F-ts, *s.* and f: $(X,\delta) \longrightarrow (Y,\gamma)$ be a mapping. if the graph g : $(X,\delta) \longrightarrow (X \times Y, \theta)$ of f is F_{μ}-perfectelycontinuous (resp., F_{μ}- completely continuous) so is f, where θ is the F- product topology generated by δ and γ

Proof. Suppose the graph g : $(X,\delta) \longrightarrow (X \times Y, \theta)$ is F_{μ}-perfectelycontinuous

Let $v \in \gamma_{f^{\rightarrow}(\mu)}$, i.e. ,$v = f^{\rightarrow}(\mu) \wedge \eta$ where $\eta \in \gamma$, we want to show that , $\mu \wedge f^{\leftarrow}(f^{\rightarrow}(\mu) \wedge \eta) \in \delta_{\mu}$. since $\underline{1} \times \eta \in \theta$, $g^{\rightarrow}(\mu) \wedge \left(\underline{1} \times \eta\right) \in \theta_{g^{\rightarrow}(\mu)}$,

then $\mu \wedge g^{\leftarrow}\left(g^{\rightarrow}(\mu) \wedge \left(\underline{1} \times \eta\right)\right) = \mu \wedge g^{\leftarrow}\left(\underline{1} \times \eta\right) = \mu \wedge (\underline{1} \wedge f^{\leftarrow}(\eta)) =$
$\mu \wedge f^{\leftarrow}(\eta) = \mu \wedge f^{\leftarrow}(f^{\rightarrow}(\mu) \wedge \eta)$ is an F_{μ}-open and an F_{μ}-closed set of $\delta\mu$

so f is F_{μ}-perfectecontinuous. The proof of F_{μ}-completely continuous by the same fashion.

Definition 1.3 [14] Let (X,δ) be a F-ts, and $A \subset X$, Then, the F- subspace (A,δ_A) is called a F_{μ}-retract of (X,δ) F_{μ} there exists a F_{μ}-continuous mapping $r : (X,\delta) \longrightarrow (A,\delta_A)$ such that r (a)= a for all a $\in A$. In this case r is called a F_{μ}-retraction.

Definition 1.4 Let (X,δ) be a F-ts, and $A \subset X$, Then, the F- subspace (A,δ_A) is called a F_{μ}-perfectely retract (F_{μ}-completely retract, F_{μ}-R-retract) of (X,δ) F_{μ} there exists a F_{μ} - F_{μ}-perfectly continuous (F_{μ}- completely continuous, F_{μ}- R- continuous) mapping $r : (X,\delta) \longrightarrow (A,\delta_A)$ such that r (a) = a for all a $\in A$. In this case r is called a F_{μ}-perfectlyretraction (F_{μ}-completelyretraction, F_{μ}-R-retractretraction)

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Remark 1.2 The implications between these different concepts are given by the following diagram:

$$\mathrm{F}_{\mu}\mathrm{P}\text{ retract } \Rightarrow \mathrm{F}_{\mu}\mathrm{C}\text{ retract } \Rightarrow \mathrm{F}_{\mu}\text{R- retract}$$

The converse of the above implication need not be true in general, as shown by the following examples.

Example 1.3. Let λ and μ be F- sets on $\mathrm{X} = \{\mathrm{a}, \mathrm{b}\}$, defined by

$$\lambda = a_{0.2} \vee b_{0.3}$$

$$\mu = a_{0.4} \vee b_{0.7}$$

,$\delta = \{\underline{0}, \underline{1}, \lambda\}$, and A= $\{\mathrm{a}\} \subset \mathrm{X}$. Then, (A, δ_A) is a F_{μ}-R-retract of

(X, δ), but not a F_{μ}- C retract.

Example 1.4 Let λ, β and μ be F- sets on $\mathrm{X} = \{\mathrm{a}, \mathrm{b}\}$, defined by

$$\lambda = a_{0.2} \vee b_{0.2}$$

$$\beta = a_{0.4} \vee b_{0.4}$$

$$\mu = a_{0.7} \vee b_{0.9}$$

$\delta = \{\underline{0}, \underline{1}, \lambda, \beta\}$, and A= $\{\mathrm{a}\} \subset \mathrm{X}$. Then, (A, δ_A) is a F_{μ}-C - retract of (X, δ), but not a F_{μ}- P- retract.

Theorem 1.4 Let (X, δ) be a F-ts , A$\subset X$ and $r : (X, \delta) \longrightarrow (A, \delta_A)$ be a mapping such that $r(a) = a \ \forall a \in A$. if the graph $g : (X, \delta) \longrightarrow (X \times A, \theta)$ of r is F_{μ}-perfectelycontinuous (resp., F_{μ}-completelycontinuos) then f is a F_{μ}-retraction , where θ is the product topology generated by δ and δ_A

Proof. It follows directly from Theorem 1.3

Definition 1.5 Let (X, δ) be a F_{μ}-ts. Then (A, δ_A) is said to be a F_{μ}-neighbourhood perfectlyretract (F_{μ}- neighborhood completely retract, F_{μ}-neighborhood R-retract) (F_{μ}-nbd P-retract, F_{μ}-nbd R-retract, F_{μ}-nbd C-retract) of (X, δ) if (A, δ_A) is a F_{μ}-perfectlyretract (F_{μ}- completelyretract, F_{μ}- R-retract) of ((Y,δ_Y) , such that A$\subset$ Y$\subset$ X, $1_Y \in \delta$

Remark 1.3 Every F_{μ}-P- retract is a F_{μ}-nbd P-retract, but the converse is

not true.

Example 1.5 Let X= $\{\mathrm{a}, \mathrm{b}, \mathrm{c}\}$, A=$\{\mathrm{a}\} \subset$ X, λ_1, λ_2 and μ be F-sets on

X, defined by

$$\lambda_1 = a_{0.2} \vee b_{0.2} \vee c_{0.4}$$

$$\lambda_2 = a_1 \vee b_1$$

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$$\mu = a_{0.4} \vee b_{0.4} \vee c_{0.5}$$

Consider $\delta = \{\underline{0}, \underline{1}, \lambda_1, \lambda_2, \lambda_1 \vee \lambda_2, \lambda_1 \wedge \lambda_2\}$. Then (A, δ_A) is a F_μ-nbd P-retract of (X, δ), but not a F_μ-P-retract of (X, δ).

Example 1.6 Let $X = \{a, b, c\}$, A= $\{a\} \subset X$, λ_1, λ_2 and μ be F- sets on X, defined by

$$\lambda_1 = a_{0.2} \vee b_{0.2} \vee c_{0.4}$$

$$\lambda_2 = a_1 \vee b_1$$

$$\mu = a_{0.8} \vee b_{0.8} \vee c_{0.5}$$

Consider $\delta = \{\underline{0}, \underline{1}, \lambda_1, \lambda_2, \lambda_1 \vee \lambda_2, \lambda_1 \wedge \lambda_2\}$. Then (A, δ_A) is a F_μ-nbd C-retract of (X, δ), but not a F_μ-C-retract of (X, δ).

Example 1.7 in example 1.6 (A, δ_A) is a F_μ-nbd R-retract of (X, δ), but not a F_μ-R-retract. of (X, δ)

2- On F_μ-semi closure and F_μ-semi-interior and on **F_μ-irresolute mapping**

Definition 2.1 Let (X, δ) be a F-ts, $\mu, \lambda \in A_\mu$. Then v *is* called

(i) [14] a F_μ-semiopen (briefly, $F_\mu so$) set if there exists

$$\lambda \in \delta_\mu \text{ . such that } \nu \leq \lambda \leq Cl_\mu(\nu) \text{ (or } \nu \leq Cl_\mu(Int_\mu(\nu)).$$

(ii) [16] a F_μ-semiclosed (briefly, $F_\mu sc$) set if there exists $\nu \in \delta_\mu$. such that

$$Int_\mu(\nu) \leq \lambda \leq \nu \text{ (or, } \lambda \leq Cl_\mu(Int_\mu(\lambda))$$

(iii) The F_μ-semi-interior of λ, denoted by $SI_\mu(\lambda) = \vee\{\nu \in \delta_\mu : \nu \leq \lambda, \ \nu \text{ is } F_\mu so\}$.

(iv) The F_μ-semi -closure of λ, denoted by $SC_\mu(\lambda) = \wedge\{\nu \in \delta_\mu : \nu \geq \lambda, \nu \text{ is} F_\mu sc\}$.

Theorem 2.1. Let (X,δ) be a F-ts, $\mu, \lambda \in \mathcal{A}_\mu$. The following statements are equivalent.

(i) λ is $F_\mu so$

(ii) $\lambda \leq Cl_\mu(Int_\mu(\lambda))$.

(iii) $Cl_\mu(\lambda) = Cl_\mu(Int_\mu(\lambda))$.

(iv) $\mu - \lambda$ is $F_\mu sc$

(v) $Int_\mu(Cl_\mu(\mu - \lambda)) \leq \mu - \lambda$

(vi) $t_\mu(Cl_\mu(\mu - \lambda)) = Int_\mu(\mu - \lambda)$

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Proof (i) ⇒ (ii) Let λ be $\mathrm{F}_{\mu}\mathrm{so}$ There exists $\nu \in \delta_{\mu}$ such that $\nu \leq \lambda \leq Cl_{\mu}(\nu)$ by Theorem 1.3. $Int_{\mu}(v) = v$ since $v \leq \lambda$, we have $Int_{\mu}(v) = v \leq Int_{\mu}(\lambda)$. It implies $Cl_{\mu}(\nu) \leq Cl_{\mu}(Int_{\mu}(\lambda)$. Since $\lambda \leq Cl_{\mu}(\nu)$, we have $\lambda \leq Cl_{\mu}(Int_{\mu}(\lambda)$.

(ii) ⇒ (iii) By the definition of Cl_{μ} and (ii), $Cl_{\mu}(\lambda) \leq Cl_{\mu}(Int_{\mu}(\lambda)$. Since, $Int_{\mu}(\lambda) \leq \lambda$, $Cl_{\mu}(Int_{\mu}(\lambda) \leq Cl_{\mu}(\lambda)$. Thus, we have $Cl_{\mu}(\lambda)$= $Cl_{\mu}(Int_{\mu}(\lambda)$.

(iii) ⇒ (i) Put $v = Int_{\mu}(\lambda)$. By the definition of t_{μ} , from Theorem 1.3, we have $\nu \leq \lambda \leq Cl_{\mu}(\lambda)$= $Cl_{\mu}(Int_{\mu}(\lambda) = Cl_{\mu}(\nu)$. Hence, λ is $\mathrm{F}_{\mu}\mathrm{so}$.

(iv) ⇒ (i) It is easily proved from the following $\nu \leq \lambda \leq Cl_{\mu}(v) \Leftrightarrow \mu - Cl_{\mu}(v) \leq \mu - \lambda \leq \mu - v \Leftrightarrow Int_{\mu}(\mu - v) \leq \mu - \lambda \leq \mu - v$. (From Theorem 1.3)

(ii) ⇒ (v) and (iii) ⇒ (vi) are easily proved from Theorem 1.3

Theorem 2.1. [14] Let (X, δ) be a F-ts, $\mu \in \mathcal{A}_{\mu}$

(i) Any union of $\mathrm{F}_{\mu}\mathrm{so}$ sets is $\mathrm{F}_{\mu}\mathrm{so}$

(ii) Any intersection of $\mathrm{F}_{\mu}\mathrm{sc}$ sets is $\mathrm{F}_{\mu}\mathrm{sc}$

Theorem 2.2. Let (X, δ) be a F-ts, μ, β, λ ∈ $\mathcal{A}_{\mu}$. Then,

(i) $Int_{\mu}(\lambda)$ is $\mathrm{F}_{\mu}\mathrm{so}$

(ii) $Cl_{\mu}(\lambda)$ is $\mathrm{F}_{\mu}\mathrm{sc}$

(iii) If λ is $\mathrm{F}_{\mu}\mathrm{so}$ and $Int_{\mu}(\lambda) \leq \beta \leq Cl_{\mu}(\lambda)$, then β is $\mathrm{F}_{\mu}\mathrm{so}$.

(iv) If λ is $\mathrm{F}_{\mu}\mathrm{sc}$ and $Int_{\mu}(\lambda) \leq \beta \leq Cl_{\mu}(\lambda)$, then β is $\mathrm{F}_{\mu}\mathrm{sc}$.

Proof we prove only (iii) and (iv).

(iii) Since λ is $\mathrm{F}_{\mu}\mathrm{so}$, then there exists $\nu \in \delta_{\mu}$ such that, $\nu \leq \lambda \leq Cl_{\mu}(\nu) \Rightarrow \nu = Int_{\mu}(\nu) \leq Int_{\mu}(\lambda)$ and $Cl_{\mu}(\lambda) \leq Cl_{\mu}(\nu)$. Thus, $\nu \leq \beta \leq Cl_{\mu}(\nu)$. Hence, β is $\mathrm{F}_{\mu}\mathrm{so}$.

(iv) It is easily proved from (iii) and Theorem 2.1. And the following

$$Int_{\mu}(\lambda) \leq \beta \leq Cl_{\mu}(\lambda) \Leftrightarrow \mu - Cl_{\mu}(\lambda) \leq \mu - \beta \leq \mu - Int_{\mu}(\lambda) \Leftrightarrow Int_{\mu}(\mu - \lambda) \leq \mu - \beta \leq Cl_{\mu}(\mu - \lambda)$$ by Theorem 2.1

Theorem 2.3. Let (X,δ) be a F-ts, $\mu, \nu, \lambda \in \mathcal{A}_{\mu}$. The following statements are valid:

(i) λ is $\mathrm{F}_{\mu}\mathrm{so}$ iff $\lambda = SI_{\mu}(\lambda)$.

(ii) λ is $\mathrm{F}_{\mu}\mathrm{sc}$ iff $\lambda = SC_{\mu}(\lambda)$.

(iii) $SC_{\mu}(\underline{0}) = \underline{0}$

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(iv) $Int_\mu(\lambda) \le SI_\mu(\lambda) \le \lambda \le SC_\mu(\lambda) \le Cl_\mu(\lambda)$.

(v) $SC_\mu(\lambda) \vee SC_\mu(\nu) = SC_\mu(\lambda \vee \nu)$.

(vi) $SC_\mu\left(SC_\mu(\lambda)\right) = SC_\mu(\lambda)$

(vii) $Cl_\mu(SC_\mu(\lambda)) = SC_\mu(Cl_\mu(\lambda)) = Cl_\mu(\lambda)$

(viii) $SI_\mu(\mu - \lambda) = \mu - SC_\mu(\lambda)$.

Proof we prove only (vii) and (viii).

(vii) From (ii) and Theorem 2.2 $SC_\mu(Cl_\mu(\lambda)) = Cl_\mu(\lambda)$, we only show that $Cl_\mu(SC_\mu(\lambda)) = Cl_\mu(\lambda)$. Since $\lambda \le SC_\mu(\lambda)$, $Cl_\mu(SC_\mu(\lambda)) \ge Cl_\mu(\lambda)$. Suppose that $Cl_\mu(SC_\mu(\lambda)) \nleq Cl_\mu(\lambda)$. By the definition of Cl_μ, there exists $\xi \in \delta_\mu$ with $\lambda \le \xi$ such that, $Cl_\mu(SC_\mu(\lambda)) \ge \xi \ge Cl_\mu(\lambda)$. On the other hand, since $\xi \le Cl_\mu(\xi)$, $\lambda \le \xi \Rightarrow SC_\mu(\lambda) \le SC_\mu(\xi) = SC_\mu(Cl_\mu(\xi)) = Cl_\mu(\xi) = \xi$. Thus, $Cl_\mu(SC_\mu(\lambda)) \le \xi$. It is a contradiction. Hence $Cl_\mu(SC_\mu(\lambda)) \le Cl_\mu(\lambda)$.

(viii) $\forall \lambda \in \delta_\mu$, we have the following: $\mu - SC_\mu(\lambda) = \mu - \wedge\{\nu : \nu \ge \lambda,\ \nu \text{ is } F_\mu sc\} = \vee\{\mu - \nu : \mu - \nu \le \mu - \lambda,\ \mu - \nu \text{ is } F_\mu so\} = SI_\mu(\mu - \lambda)$.

Definition 2.2 Let (X,δ) and (Y,γ) be a F-ts's, $\mu \in \mathcal{A}_\mu$.

Let f: $(X,\delta) \longrightarrow (Y,\gamma)$ be a mapping.

(i) [12] f is called F_μ-continuous mapping iff $f^{\leftarrow}(\upsilon) \in \delta_\mu$, for each $\upsilon \in \gamma_{f(\mu)}$.

(ii) [14] f is called F_μ-semi continuous mapping iff $f^{\leftarrow}(\upsilon)$ is $F_\mu so \in \delta_\mu$, for each $\upsilon \in \gamma_{f(\mu)}$.

(iii) f is called F_μ-irresolute mapping iff $f^{\leftarrow}(\upsilon)$ is $F_\mu so \in \delta_\mu$, for each $F_{f(\mu)}$ so $\upsilon \in \gamma_{f(\mu)}$.

(iv) f is called F_μ-irresolute open mapping iff $f(\upsilon)$ is $F_\mu so \in \gamma_{f(\mu)}$, for each $F_{f(\mu)}$ so $\upsilon \in \delta_\mu$.

(v) f is called F_μ-irresolute closed mapping iff $f(\upsilon)$ is $F_\mu sc \in \gamma_{f(\mu)}$, for each $F_{f(\mu)}$ sc $\upsilon \in \delta_\mu$.

Remark 2.1 Every F_μ-continuous mapping is F_μ- irresolute mapping, but the converse is not true.

Example 2.1 Let $X = \{a, b, c\}$, $Y = \{y\}$, $\delta = \{\underline{0}, \underline{1}, \lambda\}$. and

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$\gamma = \{ \underline{0}, \underline{1}, \theta \}$.λ and $\mu \in I^X$,$\theta \in I^Y$,defined by

$$\lambda = \quad a_{0.1} \ \vee \ b_{0.1}$$
$$\mu = \quad a_{0.2} \ \vee \ b_{0.2} \quad c_{0.3}$$
$$\theta = \quad y_{0.1}$$

Then, the constant function f is F_μ-irresolute mapping, but not F_μ-continuous.

Proposition 2.1 Let (X,δ) and (Y,γ)be a F-ts's, $\mu \in \mathcal{A}_\mu$ .Let f :$(X,\delta) \to (Y,\gamma)$ be a mapping. If f is F_μ-irresolute mapping, then for each $\mathrm{F}_\mu$sc $\lambda \in \gamma_{f(\mu)}$, $f^{\leftarrow}(\lambda)$ is $\mathrm{F}_\mu$sc $\in \delta_\mu$.

Proof For each F_μ sc set $\lambda \in \gamma_{f(\mu)} \Rightarrow f(\mu) - \lambda$ is $\mathrm{F}_\mu$so set $\in \gamma_{f(\mu)}$, $f^{\leftarrow}(f(\mu) - \lambda) \wedge \mu \leq (\mu - f^{\leftarrow}(\lambda)) \wedge \mu$ is $\mathrm{F}_\mu$so set $\in \delta_\mu$. $f^{\leftarrow}(\lambda) \wedge \mu$ is $\mathrm{F}_\mu$sc set $\in \delta_\mu$.

Proposition 2.2 Let (X,δ) and (Y,γ) be a F-ts's, $\mu \in \mathcal{A}_\mu$ .Let f :$(X,\delta) \to (Y,\gamma)$ be a mapping. If for each $\mathrm{F}_\mu$sc $\lambda \in \gamma_{f(\mu)}$, $f^{\leftarrow}(\lambda)$ is $\mathrm{F}_\mu$sc $\in \delta_\mu$ then,

$$f\left(SC_\mu(\lambda)\right) \leq SC_{f(\mu)}(f(\lambda)), \text{ for each } \lambda \in \delta_\mu.$$

Proof Suppose there exists $\lambda \in \delta_\mu$ such that, $f\left(SC_\mu(\lambda)\right) \nleq SC_{f(\mu)}(f(\lambda))$

Since, $\mathrm{SC}_{f(\mu)}(f(\lambda)) \leq \nu \in \gamma'_{f(\mu)}$. Moreover, $(f(\lambda) \leq \nu \Rightarrow \lambda \leq f^{\leftarrow}(\nu) \wedge \mu$.

$\Rightarrow f^{\leftarrow}(\nu) \wedge \mu$ is $\mathrm{F}_\mu$sc $\in \delta'_\mu$, Thus, $SC_\mu(\lambda) \leq f^{\leftarrow}(\nu) \wedge \mu \Rightarrow SC_\mu(\lambda) \leq f^{\leftarrow}(\nu) \wedge \mu \geq \lambda$, then $f(SC_\mu(\lambda)) \leq SC_{f(\mu)}(\nu \wedge f(\mu)) \geq SC_{f(\mu)}(f(\lambda))$. It is a contradiction

Proposition 2.3 Let (X,δ) and (Y,γ) be a F-ts's, $\mu \in \mathcal{A}_\mu$. Let f: $(X,\delta) \longrightarrow (Y,\gamma)$ be a mapping. If $f^{\leftarrow}(SI_{f(\mu)}(\lambda)) \wedge \mu \leq SI_\mu(f^{\leftarrow}\lambda) \wedge \mu)$, for each λ is

$\mathrm{F}_\mu$so $\in \gamma_{f(\mu)} \in \gamma_{f(\mu)}$, then f is F_μ-irresolute mapping.

Proof Let λ is $\mathrm{F}_\mu$so $\in \gamma_{f(\mu)}$ From theorem 2.3(i). $\lambda = SI_{f(\mu)}(\lambda)$. Since, $f^{\leftarrow}(\lambda)) \wedge \mu \leq SI_\mu(f^{\leftarrow}(\lambda)) \wedge \mu)$. On the other hand, by Theorem 2.3(iv), $f^{\leftarrow}(\lambda)) \wedge \mu \geq SI_\mu(f^{\leftarrow}(\lambda) \wedge \mu)$. Thus, $f^{\leftarrow}(\lambda)) \wedge \mu = SI_\mu(f^{\leftarrow}(\lambda)) \wedge \mu)$, that is $f^{\leftarrow}(\lambda)) \wedge \mu$ is $\mathrm{F}_\mu$so $\in \delta_\mu \Rightarrow f$ is F_μ-irresolute mapping.

Theorem 2.4 Let (X,δ) and (Y,γ) be a F-ts's , $\mu \in \mathcal{A}_\mu$. Let f : $(X,\delta) \longrightarrow (Y,\gamma)$ be a mapping. The following statements are equivalent.

(I) A map f is F_μ -irresolute open mapping

(ii) $f(SI_\mu(\lambda)) \wedge f(\mu) \leq SI_{f(\mu)}(f(\lambda) \wedge f(\mu))$, for each λ is $\mathrm{F}_\mu$so $\in \delta_\mu$.

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(iii) $SI_\mu(f^{\leftarrow}\lambda) \wedge \mu \leq (f^{\leftarrow} (SI_{f(\mu)}(\lambda)) \wedge \mu$, for each $\lambda \in \gamma_{f(\mu)}$

(iv) For any $\nu \in \gamma'_{f(\mu)}$ and any $F_\mu sc$ $\lambda \in \delta_\mu$ such that $f^{\leftarrow}(\nu) \wedge \mu \leq \lambda$, there exists $F_\mu sc$ set $\rho \in \gamma_{f(\mu)}$ with $\nu \leq \rho$ such that $f^{\leftarrow}(\rho) \wedge \mu \leq \lambda$.

Proof (i) ⇒ (ii) For each λ be $F_\mu so$ set $\in \delta_\mu$, since $SI_\mu(\lambda) \leq \lambda$ from Theorem 2.3(iv). $f(SI_\mu(\lambda)) \wedge f(\mu) \leq f(\lambda) \wedge f(\mu)$ by (i) $f(SI_\mu(\lambda)) \wedge f(\mu)$ is F_μ so set $\in \delta_\mu$, hence, $f(SI_\mu(\lambda)) \wedge f(\mu) \leq SI_{f(\mu)}(f(\lambda) \wedge f(\mu))$.

(ii) ⇒ (iii) for each $\lambda \in \gamma_{f(\mu)}$ from (ii) $f(SI_\mu(f^{\leftarrow}(\lambda))) \wedge f(\mu) \leq SI_{f(\mu)}(f(f^{\leftarrow}(\lambda)) \wedge$

$$f(\mu)) \leq SI_{f(\mu)}(\lambda) \wedge f(\mu) \Rightarrow SI_\mu(f^{\leftarrow}(\lambda))) \wedge \mu \leq f^{\leftarrow} (SI_{f(\mu)}(\lambda)) \wedge \mu.$$

(iii) ⇒ (iv) Let λ be F_μ sc set $\in \delta'_\mu$ and $\lambda \in \gamma'_{f(\mu)}$ such that $f^{\leftarrow}(\nu) \wedge \mu \leq \lambda$.

from Theorem 2.2 $\lambda = Int_\mu (CI_\mu((\lambda))$. Since $\mu - \lambda = f^{\leftarrow} (\mu - \nu) \wedge \mu$, we have $SI_\mu (\mu - \lambda) = \mu - \lambda \leq SI_\mu\left(f^{\leftarrow} (\mu - \nu)\right) \wedge \mu$, by (iii) $\mu - \lambda \leq SI_\mu\left(f^{\leftarrow} (\mu - \nu)\right) \wedge \mu \leq f^{\leftarrow} (SI_{f(\mu)} (\mu - \nu)) \wedge \mu \Rightarrow \lambda \geq \mu - \left(f^{\leftarrow}\left(SI_{f(\mu)}(\mu - v)\right) \wedge \mu\right) = f^{\leftarrow}\left(\mu - (SI_{f(\mu)}(\mu - v)\right) \wedge \mu) = f^{\leftarrow}(SC_{f(\mu)}(v)) \wedge \mu$.

By Theorem 2.3 (viii), thus there exists F_μ sc set $\rho = SC_{f(\mu)}(v) \in \gamma'_{f(\mu)}$ with $v \leq \rho$ such that $f^{\leftarrow}(\rho) \wedge \mu \leq \lambda$

(iv) ⇒ (i) Let σ be F_μ so set $\in \delta_\mu$, $\lambda = \mu - \sigma$ is F_μ sc set $\in \delta'_\mu$ put $v = f(\mu) - f(\sigma) \in \gamma'_{f(\mu)}$ we obtain $f^{\leftarrow}(v) \wedge \mu = f^{\leftarrow}(f(\mu) - f(\sigma)) \leq \mu - (\sigma)) = \lambda$. by (iv) there exists $\rho \in \gamma'_{f(\mu)}$ with $v \leq \rho$ such that $f^{\leftarrow}(\rho) \wedge \mu \leq \lambda = \mu - \sigma \Longrightarrow \sigma = \mu - (f^{\leftarrow}(\rho) \wedge \mu) = f^{\leftarrow}(\mu - \rho) \wedge \mu$, Thus $f(\sigma) \wedge f(\mu) \leq f(f^{\leftarrow}(\mu - \rho) \wedge \mu) \leq (\mu - \rho) \wedge f(\mu))$ (1)

On the other hand, since $v \leq \rho$ From (1)

$f(\sigma) \wedge f(\mu) = f(\mu) - v \geq f(\mu) - \rho$. Hence, $f(\sigma) \wedge f(\mu) = f(\mu) - \rho$ that is $f(\sigma)$ is F_μ so $\in \gamma_{f(\mu)}$. Then f is F_μ-irresolute open mapping

Definition 2.3 Let (X, δ) and (Y, γ) be a F- ts's, $\mu \in \mathcal{A}_\mu$. Let f: (X, δ) → (Y, γ) be a mapping, then f is called F_μ-almost open mapping iff for each $\lambda \in \delta_\mu$, with $\lambda = Int_\mu(Cl_\mu(\lambda))$. $f(\lambda) \in \gamma_{f(\mu)}$.

Theorem 2 .5 2 Let (X, δ) and (Y, γ) be a F- ts's, $\mu \in \mathcal{A}_\mu$. Let $f : (X, \delta) \to (Y, \gamma)$ be a mapping. The following statments are equivalent .

(i) A map f is F_μ-almost open mapping

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(ii) $f\,(Int_{\mu}(\lambda)) \leq\ Int_{f(\mu)}\left(f(\lambda)\right)$, for each λ is F_{μ} sc $\in \delta_{\mu}$

(iii) For any $\upsilon \in \gamma^{c}_{f(\mu)}$ and any $\lambda = Cl_{\mu}(Int_{\mu}(\lambda))$ such that $f^{\leftarrow}(\upsilon) \wedge \mu \leq \lambda$ there exists $\rho\ \in\ \gamma_{f(\mu)}$ and $\upsilon \leq \rho$ such that $f^{\leftarrow}(\rho) \wedge \mu \leq \lambda$

Proof (i) $\Rightarrow$ (ii) Let λ be F_{μ} sc $\in \delta_{\mu}$ that is $Int_{\mu}(Cl_{\mu}(\lambda))\ \leq\ \lambda$. From Theorem 2.2, we easily prove the following $Int_{\mu}(Cl_{\mu}(\lambda)) = Int_{\mu}(Cl_{\mu}(\ Cl_{\mu}(\lambda)))$.

Since f is F_{μ}-almost open mapping, $Int_{f(\mu)}(f\ (Int_{\mu}(Cl_{\mu}(\lambda)) = f(\ Int_{\mu}(Cl_{\mu}(\lambda)) \in \gamma_{f(\mu)}$. (1)

On the other hand, $Int_{\mu}\left(Cl_{\mu}(\lambda)\right) \leq\ \lambda\ \Rightarrow\ Int_{\mu}(Int_{\mu}(Cl_{\mu}(\lambda))) \leq Int_{\mu}(\lambda)$,

Thus, $Int_{\mu}(\lambda) =\ Int_{\mu}(Cl_{\mu}(\lambda)) \leq\ \lambda. \Rightarrow f\left(\ Int_{\mu}(\lambda)\right) = f(Int_{\mu}(Cl_{\mu}(\lambda))) =$

$Int_{f(\mu)}(f\ (Int_{\mu}(Cl_{\mu}(\lambda)))\ \leq\ Int_{f(\mu)}(f(\lambda))$ From (1)

(ii) $\Rightarrow$ (i) $\lambda =\ Int_{\mu}\left(Cl_{\mu}(\lambda)\right) \in \delta_{\mu}$. Since $Int_{\mu}(\lambda) =\ \lambda$ and λ is F_{μ} sc by (ii),

$f\,(\lambda\,) = f\left(Int_{\mu}(\lambda)\right) \leq\ Int_{f(\mu)}\left(f(\lambda)\right)$ From Theorem 2.2,

$f(\lambda) =\ Int_{f(\mu)}\left(f(\lambda)\right) \in\ \gamma_{f(\mu)}$.

(i) $\Rightarrow$ (iii) let $\lambda = Cl_{\mu}(Int_{\mu}(\lambda))$ and $\upsilon \in \gamma^{c}_{f(\mu)}$ such that $f^{\leftarrow}(\upsilon) \wedge \mu \leq \lambda$. But

$\rho = f(\mu) -\ f(\mu - \lambda)$ since $\mu - \lambda = Int_{\mu}\left(Cl_{\mu}(\mu - \lambda)\right)$, by (1). Since $f^{\leftarrow}(\upsilon) \wedge \mu \leq \lambda$ iff $\upsilon \leq\ f(\mu) - f(\mu - \lambda)$ then, $\upsilon \leq \rho$, also, $f^{\leftarrow}(\rho) \wedge \mu =\ f^{\leftarrow}(f(\mu) -\ f(\mu - \lambda)) \leq\ \mu - (\mu - \lambda)) =\ \lambda\ \Longrightarrow\ \ f^{\leftarrow}(\rho) \wedge \mu \leq\ \lambda$.

(iii) $\Longrightarrow$ (i) let σ be F_{μ} $sc \in \delta^{c}_{\mu}$ such that $\sigma = Int_{\mu}(Cl_{\mu}(\sigma))$

put $\upsilon = f(\mu) -\ f(\sigma)$ and $\lambda\ =\ \mu - \sigma$ with $\lambda = Cl_{\mu}(Int_{\mu}(\lambda))$, we obtain

$f^{\leftarrow}(\upsilon)\ \wedge\ \mu =\ f^{\leftarrow}\left(f(\mu) - f(\sigma)\right)\ \leq \mu - (\sigma)) =\ \lambda$ by (iii) there exists $\rho \in\ \gamma_{f(\mu)}$

with $\upsilon \leq\ \rho$ such that $f^{\leftarrow}(\rho)\ \wedge\ \mu = \leq\ \lambda = \mu - \sigma \Rightarrow\ \sigma = \mu - (f^{\leftarrow}(\rho)\ \wedge\ \mu)\ =$ $f^{\leftarrow}(\mu - \rho)\ \wedge\ \mu$, Thus $f(\sigma) \wedge f(\mu) \leq f(f^{\leftarrow}(\mu - \rho) \wedge\ \mu) \leq (\mu - \rho) \wedge f(\mu))$ (1)

On the other hand, since $\upsilon \leq \rho$ From (1)

$f(\sigma) \wedge f(\mu) = f(\mu) - \upsilon \geq (\mu - \rho) \wedge f(\mu))$ (2)

Hence from (1) and (2) $f(\sigma) \wedge f(\mu) = (\mu - \rho)\ \wedge f(\,\mu)$

Theorem 2.6 Let $(\,X,\delta)$ and $(\,Y,\gamma\,)$ be a F-ts's, $\mu\ \in\ \mathcal{A}_{\,\mu}$. Let $f\ :\ (\,X,\delta)\ \longrightarrow (Y,\gamma\,)$ is F_{μ}- semi continuous and F_{μ}-almost open mapping, then f is F_{μ}-irresolute mapping

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Proof By Proposition 2.1, we will show that $f^{\leftarrow}(\lambda) \wedge \mu$ is F_{μ} sc set,

$\forall\, \mathrm{F}_{\mu}$ − sc set $\lambda \in \gamma_{f(\mu)}$. Since λ is $\mathrm{F}_{f(\mu)}$ sc set $\in \gamma_{f(\mu)}$, we have $Int_{f(\mu)}\,(Cl_{f(\mu)}\,(\lambda) \leq \lambda$. Since f is F_{μ} − semi Continuous mapping, $f^{\leftarrow}\,(f(\mu) - Cl_{f(\mu)}(\lambda)) \wedge \mu =$ $(\mu\, f^{\leftarrow}\left(C\, l_{f(\mu)}(\lambda)\right) \wedge \mu$ is F_{μ} sc set $\in \delta_{\mu}$, that is $f^{\leftarrow}\left(C\, l_{f(\mu)}(\lambda)\right) \wedge \mu$ is F_{μ} sc set $\in \delta_{\mu}^{c}$ so, $Int_{\mu}\,(Cl_{\mu}(f^{\leftarrow}\left(C\, l_{f(\mu)}(\lambda)\right) \wedge \mu \leq f^{\leftarrow}\left(C\, l_{f(\mu)}(\lambda)\right) \wedge \mu \Rightarrow Int_{\mu}\,(Cl_{\mu}(f^{\leftarrow}\left(C\, l_{f(\mu)}(\lambda)\right) \wedge$

$$\leq Int_{\mu}\,((f^{\leftarrow}\left(C\, l_{f(\mu)}(\lambda)\right) \wedge \mu \quad (1)$$

since f is F_{μ}-almost open mapping, and λ is F_{μ} sc set $\gamma_{f(\mu)}$. By propostion 2.2

$$f\,(Int_{\mu}((f^{\leftarrow}\left(C\, l_{f(\mu)}(\lambda)\right) \wedge \mu) \leq Inl_{f(\mu)}(\mathrm{f}\,(f^{\leftarrow}\left(C\, l_{f(\mu)}(\lambda)\right) \wedge \mu\,)\;\; Inl_{f(\mu)}\,(Cl_{f(\mu)}(\lambda) =$$

$$Inl_{f(\mu)}\,(\lambda) \leq \lambda. \quad \Rightarrow Int_{\mu}\,((f^{\leftarrow}\left(C\, l_{f(\mu)}(\lambda)\right) \wedge \mu \leq f^{\leftarrow}(\lambda) \wedge \mu \;(2),$$

Thus, we have $Int_{\mu}\,(Cl_{\mu}(f^{\leftarrow}\,(\lambda)) \wedge \mu) \leq Int_{\mu}\,(Cl_{\mu}(f^{\leftarrow}\left(C\, l_{f(\mu)}(\lambda)\right) \wedge \mu) \leq Int_{\mu}\,(f^{\leftarrow}\left(C\, l_{f(\mu)}(\lambda)\right) \wedge \mu)$ by (1) $\leq f^{\leftarrow}\,(\lambda) \wedge \mu$ by (2). Hence $f^{\leftarrow}\,(\lambda)$ *is* F_{μ} sc.

CONCLUSION

The purpose of this paper is to define and introduced anther applications of fuzzy continuity using the definition of Rod bough [J. Math. Anal. Appl. 79 (1981) 273].

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