Xi'an Shiyou Daxue Xuebao (Ziran Kexue Ban)/
Journal of Xi'an Shiyou University, Natural Sciences Edition
ISSN: 1673-064X
E-Publication: Online Open Access
Vol: 66 Issue 05 | 2023
DOI 10.17605/OSF.IO/YWEC2

CONNECTEDNESS AND COMPACTNESS VIA INTUITIONISTIC SEMI * OPEN SETS

G. ESTHER RATHINAKANI
Research Scholar, PG and Research Department of Mathematics, Kamaraj College, Thoothukudi, Tamilnadu, India. Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelvei, Tamilnadu, India. Reg. No. 19222102092011, E-mail: estherrathinakani@gmail.com

M. NAVANEETHAKRISHNAN
Associate Professor, PG and Research Department of Mathematics, Kamaraj College, Thoothukudi, Tamilnadu, India. Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelvei, Tamilnadu, India. E-mail: navaneethan65@yahoo.co.in

Abstract

In this article certain kinds of intuitionistic semi * connectedness and intuitionistic semi * compactness are defined in intuitionistic topological space and their characteristics are investigated. Here we introduce intuitionistic semi * connectedness, intuitionistic semi * C_i- connectedness (i = 1,2,3,4,5), intuitionistic semi * compactness and obtain many properties.

Index Terms: intuitionistic semi * connectedness, intuitionistic semi * C_i- connectedness, intuitionistic semi * compactness intuitionistic semi * open, intuitionistic semi * closed, IS*O, IS*C.

1. INTRODUCTION

Atanassov [6] is the person who first presented the idea of intuitionistic set. After that this concept is generalized to intuitionistic sets in [1], [2] and intuitionistic topological spaces in [3]. An idea of intuitionistic connectedness and intuitionistic compactness in intuitionistic topological space is given in [5]. In this article we establish the concepts of intuitionistic semi * connectedness, intuitionistic semi * C_i- connectedness, intuitionistic semi * compactness, intuitionistic semi * lindelof spaces. Also we encounter their basic properties and explore their relationship with already existing concepts.

2. PRIME NEEDS

Definition 2.1. Let X be a nonempty fixed set. An intuitionistic set (IS in short) Ã is an object having the form Ã = < X, $^{(1)}$, $^{(2)}$ > where $^{(1)}$ and $^{(2)}$ are subsets of X such that $A^{(1)} \cap A^{(2)} = \emptyset$. The set $^{(1)}$ is called the set of member of Ã, while $A^{(2)}$ is called the set of non-member of Ã.

Definition 2.2. Let X be a non empty set, Ã = < X, $^{(1)}$, $^{(2)}$ > and B̃ = < X, $^{(1)}$, $^{(2)}$ > be an IS sets and let { Ã$_i$: $i \in j$} be arbitrary family of IS, where Ã$_i$ = < X, $A_i^{(1)}$, $A_i^{(2)}$ >.

Xi'an Shiyou Daxue Xuebao (Ziran Kexue Ban)/
Journal of Xi'an Shiyou University, Natural Sciences Edition
ISSN: 1673-064X
E-Publication: Online Open Access
Vol: 66 Issue 05 | 2023
DOI 10.17605/OSF.IO/YWEC2

Then the following results are hold.

i. $\tilde{A} \subseteq \tilde{B}$ if and only if $^{(1)} \subseteq {}^{(1)}$ and $^{(2)} \subseteq {}^{(2)}$.
ii. $\tilde{A} = \tilde{B}$ if and only if $\tilde{A} \subseteq \tilde{B}$ and $\tilde{B} \subseteq \tilde{A}$.
iii. $\overline{\tilde{A}} = < X, {}^{(2)}, {}^{(1)} >$ is called the complement of $\tilde{A}$. It is aslo denoted by $X - \tilde{A}$.
iv. $\cup \tilde{A}_i = < X, \cup A_i^{(1)}, \cap A_i^{(2)} >$.
v. $\cap \tilde{A}_i = < X, \cap A_i^{(1)}, \cup A_i^{(2)} >$.
vi. $\tilde{A} - \tilde{B} = \tilde{A} \cap \overline{\tilde{B}}$.
vii. $\widetilde{\emptyset}_I = < X, \emptyset, X >$ and $\widetilde{X}_I = < X, X, \emptyset >$.

Definition 2.3. An intuitionistic topology (IT in short) by subsets of a nonempty set X is a family τ of IS's satisfying the following axioms.

(a) $\widetilde{\emptyset}_I$, $\widetilde{X}_I \in \tau$
(b) $\tilde{G}_1 \cap \tilde{G}_2 \in \tau$ for every $\tilde{G}_1$, $\tilde{G}_2 \in \tau$
(c) $\cup \tilde{G}_i \in \tau$ for any arbitrary family $\{ \tilde{G}_i : i \in J\} \subseteq \tau$.

The pair (X, τ) is called an intuitionistic topological space (ITS in short) and any IS $\tilde{A}$ in τ is called an intuitionistic open set (IOS). The complement of an IOS $\tilde{A}$ in τ is called an intuitionistic closed set (ICS)

Definition 2.4. Let X be a nonempty set and p ∈ X be a fixed element. Then the IS $\tilde{p}_I$ (resp. p_{IV}) defined by $\tilde{p}_I =< X, \{p\}, \{p\}^c >$ (resp. $\tilde{p}_{IV} =< X, \emptyset, \{p\}^c >$ is called an intuitionistic point (resp. intuitionistic vanishing point).

Definition 2.5. Let (X, τ) be an ITS and $\tilde{A} =< X, A^1, A^2 >$ be an IS in X, $\tilde{A}$ is said to be intuitionistic generalized closed set (briefly Ig – closed set) $Icl(\tilde{A}) \subseteq \tilde{U}$ whenever $\tilde{A} \subseteq \tilde{U}$ and $\tilde{U}$ is IO in X.

Definition 2.6. If $\tilde{A}$ is an IS of an ITS (X, τ), then the intuitionistic generalized closure of $\tilde{A}$ is is denoted by $Icl^*(\tilde{A})$ and is defined as $Icl^*(\tilde{A}) = \{\tilde{E} : \tilde{E}$ is Ig – closed set and $\tilde{A} \subseteq \tilde{E}\}$.

Definition 2.7.

i. Intuitionistic semi * open sets if there is an intuitionistic open set $\tilde{G}$ in X such that $\tilde{G} \subseteq \tilde{A} \subseteq Icl^*(\tilde{G})$.
ii. intuitionistic semi * closed set if X - $\tilde{A}$ is intuitionistic semi * open.

Definition 2.8. The intuitionistic semi * interior of $\tilde{A}$ is defined as the union of all intuitionistic semi * open sets of X contained in $\tilde{A}$. It is denoted by IS*int $(\tilde{A})$.

Definition 2.9. The semi * closure of an IS $\tilde{A}$ is defined as the intersection of all intuitionistic semi * closed sets in X that containing $\tilde{A}$. It is denoted by IS*cl$(\tilde{A})$.

ISSN: 1673-064X
E-Publication: Online Open Access
Vol: 66 Issue 05 | 2023
DOI 10.17605/OSF.IO/YWEC2

Theorem 2.10. Let (X, τ$_I$) be an ITS and $\widetilde{A}$ be any ITS. Then

i. $\widetilde{A}$ is intuitionistic semi * regular if and only if IS $*$Fr($\widetilde{A}$)= $\widetilde{\emptyset}_I$.

ii. IS $*$Fr($\widetilde{A}$) = IS $*$cl($\widetilde{A}$) ∩ IS $*$cl(X − $\widetilde{A}$).

Definition 2.11. The function f: (X, τ$_1$) → (Y, τ$_2$) is said to be intuitionistic semi * continuous (summarizing IS*-Cts) if $f^{-1}(\widetilde{U})$ is IS*O in (X, τ$_1$)for every IOS $\widetilde{U}$ in (Y, τ$_2$).

Definition 2.12. Two IS's $\tilde{E}$ and $\tilde{F}$ are said to be overlapping if $\tilde{E} \not\subseteq$ X− $\tilde{F}$. Conversely $\tilde{E}$ and $\tilde{F}$ are said to be nonoverlapping, if $\tilde{E} \subseteq$ X − $\tilde{F}$. Notice that $\tilde{E} \not\subseteq$ X − $\tilde{F}$ if and only if $E^{(1)} \not\subseteq F^{(1)}$ or $\tilde{}\, E^{(1)} \not\supseteq F^{(2)}$.

3. INTUITIONISTIC SEMI * CONNECTED

Definition 3.1. An ITS (X, τ) is said to be an intuitionistic semi * connected if $\widetilde{X}_I$ cannot be expressed as the union of two disjoint nonempty IS*O sets in X.

Theorem 3.2. Every intuitionistic semi * connected is intuitionistic connected.

Proof. Let X be an intuitionistic semi * connected. To prove X is an intuitionistic connected. Suppose X is not an intuitionistic connected. Then there exist a disjoint nonempty IOS $\widetilde{U}$ and $\widetilde{V}$ such that $\widetilde{X}_I = \widetilde{U} \cup \widetilde{V}$. Since $\widetilde{U}$ and $\widetilde{V}$ are IOS, both $\widetilde{U}$ and $\widetilde{V}$ are IS*O. This is a contradiction to X is an intuitionistic semi * connected. Hence X is an intuitionistic connected.

Remark 3.3. The converse of the above theorem need not be true as shown in the succeeding example

Example 3.4. Let X = {i, j, k} and τ = {$\widetilde{X}_I$, $\widetilde{\emptyset}_I$, < X, {j}, {i, k} >, < X, {i}, {j} >, < X, {i, j}, ∅ >}. Then IS*O(X, τ) = {$\widetilde{X}_I$, $\widetilde{\emptyset}_I$, < X, {j}, {i, k} >, < X, {i}, {j} >, < X, {i, j}, ∅ >, < X, {i, k}, {j} >}. Clearly X is an intuitionistic connected but not an intuitionistic semi * connected.

Theorem 3.5. Every intuitionistic semi connected is intuitionistic semi * connected.

Proof. Let X be an intuitionistic semi connected. To prove X is an intuitionistic semi * connected. Suppose X is not an intuitionistic semi * connected. Then there exist a disjoint nonempty IS*O sets $\widetilde{U}$ and $\widetilde{V}$ such that $\widetilde{X}_I = \widetilde{U} \cup \widetilde{V}$. Since $\widetilde{U}$ and $\widetilde{V}$ are IS*O, both $\widetilde{U}$ and $\widetilde{V}$ are ISO sets. This is a contradiction to X is an intuitionistic semi connected. Hence X is an intuitionistic semi *connected.

Remark 3.6. The converse of the above theorem need not be true as shown in the succeeding example.

Example 3.7. Let X = {i, j, k} and τ = {$\widetilde{X}_I$, $\widetilde{\emptyset}_I$, < X, {i}, {j, k} >, < X, {k}, {i, j} >, < X, {i, k}, {j} >}. Then IS*O(X, τ) = {$\widetilde{X}_I$, $\widetilde{\emptyset}_I$, < X, {i}, {j, k} >, < X, {k}, {i, j} >, < X, {i, k}, {j} >, < X, {i}, {k} >, < X, {k}, {i} >, < X, {i. k}, ∅>}. Then X is an intuitionistic semi * connected but not an intuitionistic semi connected.

Xi'an Shiyou Daxue Xuebao (Ziran Kexue Ban)/
Journal of Xi'an Shiyou University, Natural Sciences Edition
ISSN: 1673-064X
E-Publication: Online Open Access
Vol: 66 Issue 05 | 2023
DOI 10.17605/OSF.IO/YWEC2

Theorem 3.8. An ITS (X, τ) has the only intuitionistic semi * regular subsets are $\widetilde{\emptyset}_I$ and $\widetilde{X}_I$ itself then (X, τ) is an intuitionistic semi * connected.

Proof. Assume that $\widetilde{\emptyset}_I$ and $\widetilde{X}_I$ are the only intuitionistic semi * regular subsets of X. To prove X is an intuitionistic semi * connected. Suppose X is not an intuitionistic semi * connected. Then there exist a disjoint nonempty IS*O sets $\widetilde{A}$ and $\widetilde{B}$ such that $\widetilde{X_I} = \widetilde{A} \cup \widetilde{B}$. Therefore $\widetilde{A} = X - \widetilde{B}$ is IS*C. Hence $\widetilde{A}$ is an intuitionistic semi * regular which is contradiction to our assumption. Hence X is an intuitionistic semi * connected.

Theorem 3.9. An ITS is an intuitionistic semi * connected if and only if every nonempty proper subsets of X has nonempty intuitionistic semi * frontier.

Proof. Let X be an intuitionistic semi * connected and $\tilde{A}$ be any nonempty IS of X. To prove IS*Fr($\tilde{A}$) ≠ $\widetilde{\emptyset}_I$. Suppose IS*Fr($\tilde{A}$) = $\widetilde{\emptyset}_I$. Then by theorem 2.10, $\tilde{A}$ is an intuitionistic semi * regular. Now by theorem 3.8, $\tilde{A}$ is not an intuitionistic semi * connected. This is a contradiction to our hypothesis. Therefore IS*Fr($\tilde{A}$) ≠ $\widetilde{\emptyset}_I$. Conversely, assume that $\tilde{A}$ is any nonempty IS of X such that IS*Fr($\tilde{A}$) ≠ $\widetilde{\emptyset}_I$. To prove X is an intuitionistic semi * connected. Suppose X is not an intuitionistic semi * connected. Then there exist a nonempty IS*O sets $\widetilde{U}$ and $\widetilde{V}$ such that $\widetilde{X}_I = \widetilde{U} \cup \widetilde{V}$. Therefore $\widetilde{U} = X - \widetilde{V}$. Hence $\widetilde{U}$ is both IS*O and IS*C. Therefore by theorem 2.10, IS*Fr($\tilde{A}$) = $\widetilde{\emptyset}_I$ which is a contradiction to our assumption. Thus X is an intuitionistic semi * connected.

Theorem 3.10. Let (X, τ_1) and (Y, τ_2) be the two ITS and f: X → Y be the surjection map, intuitionistic semi * continuous and X be an intuitionistic semi * connected. Then Y is an intuitionistic semi * connected.

Proof. Let f: X → Y be the surjection, intuitionistic semi * continuous and X be an intuitionistic semi * connected. Assume that Y is not an intuitionistic semi * connected thats lead us to there exist a disjoint nonempty IS*O sets $\widetilde{U}$ and $\widetilde{V}$ such that $\widetilde{Y}_I = \widetilde{U} \cup \widetilde{V}$. Since f is an IS*-Cts, $f^{-1}(\widetilde{U})$ and $f^{-1}(\widetilde{V})$ is IS*O in X. Since $\widetilde{U} \neq \widetilde{\emptyset}_I$ and $\widetilde{V} \neq \widetilde{\emptyset}_I$, $f^{-1}(\widetilde{U}) \neq \widetilde{\emptyset}_I$ and $f^{-1}(\widetilde{V}) \neq \widetilde{\emptyset}_I$. We have $\widetilde{Y}_I = \widetilde{U} \cup \widetilde{V}$ implies $f^{-1}(\widetilde{Y}_I) = f^{-1}(\widetilde{U}) \cup f^{-1}(\widetilde{V})$. Therefore $\widetilde{X}_I = f^{-1}(\widetilde{U}) \cup f^{-1}(\widetilde{V})$ and $f^{-1}(\widetilde{U}) \cap f^{-1}(\widetilde{V}) = f^{-1}(\widetilde{U} \cap \widetilde{V}) = f^{-1}(\widetilde{\emptyset}_I) = \widetilde{\emptyset}_I$. Therefore (X, τ_1) is not an intuitionistic semi * connected. This is a contradiction to our hypothesis. Hence (Y, τ_2) is an intuitionistic semi * connected.

Theorem 3.11. Let (X, τ_1) and (Y, τ_2) be the two ITS and f: X → Y be an injection map, IPS*O and IPS*C. If Y is an intuitionistic semi * connected, then X is an intuitionistic semi * connected.

Proof. Assume (X, τ_1) is not an intuitionistic semi * connected thats lead us to there exist a nonvoid IS*O sets $\widetilde{U}$ and $\widetilde{V}$ such that $\widetilde{Y}_I = \widetilde{U} \cup \widetilde{V}$ and $\widetilde{U} \cap \widetilde{V} = \widetilde{\emptyset}_I$. Then $\widetilde{U} = X - \widetilde{V}$.Therefore $\widetilde{U}$ is both IS*O and IS*C in X. We have f: X → Y is both IPS*O and IPS*C, $f^{-1}(\widetilde{U})$ is both IS*O and IS*C in Y . Therefore by theorem 2.10 IS $*$ Fr($f^{-1}(\widetilde{U})$) = $\widetilde{\emptyset}_I$. Thus by theorem 3.9, Y is not an intuitionistic semi * connected which is contradiction. Hence (X, τ_1) is an intuitionistic semi * connected.

ISSN: 1673-064X
E-Publication: Online Open Access
Vol: 66 Issue 05 | 2023
DOI 10.17605/OSF.IO/YWEC2

Theorem 3.12. Let (X, τ_1) and (Y, τ_2) be the two ITS and f: X → Y is an IS*O and IS*C injection map and (Y, τ_2) is an intuitionistic semi * connected, then (X, τ_1) is an intuitionistic connected.

Proof. Assume (X, τ_1) is not an intuitionistic connected thats lead us to there exist a nonempty IO sets $\widetilde{U}$ and $\widetilde{V}$ such that $\widetilde{Y_I} = \widetilde{U} \cup \widetilde{V}$ and $\widetilde{U} \cap \widetilde{V} = \widetilde{\emptyset}_I$. Then $\widetilde{U} = X - \widetilde{V}$.Therefore $\widetilde{U}$ is both IOS and ICS in X. Then $\widetilde{U}$ is both IS*O and IS*C. Since f is both IS*O and IS*C, $f(\widetilde{U})$ is an intuitionistic semi * regular in Y. Therefore by theorem 2.10, IS $*$Fr(f($(\widetilde{U})$) $= \widetilde{\emptyset}_I$. Thus by theorem 3.9, Y is not an intuitionistic semi * connected which is contradiction. Thus (X, τ_1) is an intuitionistic connected.

Definition 3.13. Let (X, τ) be an ITS and $\widetilde{U}$ be any IS of X. If there exist IS*O sets $\tilde{A}$ and $\tilde{B}$ in X satisfying the following properties, then $\widetilde{U}$ is called intuitionistic semi * C_i- disconnected.

(i) C_1 : $\widetilde{U} \subseteq \tilde{A} \cup \tilde{B}$, $\tilde{A} \cap \tilde{B} \subseteq X - \widetilde{U}$, $\widetilde{U} \cap \tilde{A} \neq \widetilde{\emptyset}_I$, $\widetilde{U} \cap \tilde{B} \neq \widetilde{\emptyset}_I$.

(ii) C_2: $\widetilde{U} \subseteq \tilde{A} \cup \tilde{B}$, $\widetilde{U} \cap \tilde{A} \cap \tilde{B} = \widetilde{\emptyset}$, $\widetilde{U} \cap \tilde{A} \neq \widetilde{\emptyset}_I$, $\widetilde{U} \cap \tilde{B} \neq \widetilde{\emptyset}_I$.

(iii) C_3: $\widetilde{U} \subseteq \tilde{A} \cup \tilde{B}$, $\tilde{A} \cap \tilde{B} \subseteq X - \widetilde{U}$, $\tilde{A} \nsubseteq X - \widetilde{U}$, $\tilde{B} \nsubseteq X - \widetilde{U}$.

(iv) C_4: $\widetilde{U} \subseteq \tilde{A} \cup \tilde{B}$, $\widetilde{U} \cap \tilde{A} \cap \tilde{B} = \widetilde{\emptyset}$, $\tilde{A} \subseteq X - \widetilde{U}$, $\tilde{B} \subseteq X - \widetilde{U}$.

Definition 3.14. Let (X, τ) be an ITS and $\widetilde{U}$ be any IS of X. If $\widetilde{U}$ is said to be an intuitionistic semi * C_i- connected, then $\widetilde{U}$ is not an intuitionistic semi * C_i- disconnected where i = 1,2,3,4.

Theorem 3.15. Let (X, τ) be an ITS and $\widetilde{U}$, $\widetilde{V}$ be any two IS of X. If $\widetilde{U}$, $\widetilde{V}$ are intuitionistic semi * C_1- connected and $\widetilde{U} \cap \widetilde{V} \neq \widetilde{\emptyset}_I$, then $\widetilde{U} \cup \widetilde{V}$ is also an intuitionistic semi * C_1- connected.

Proof. Let $\widetilde{U}$, $\widetilde{V}$ be intuitionistic semi * C_1- connected. Suppose $\widetilde{U} \cup \widetilde{V}$ is not an intuitionistic semi * C_1- connected. Then there exist an IS*O set $\tilde{C}$ and $\widetilde{D}$ such that $\widetilde{U} \cup \widetilde{V} \subseteq \tilde{C} \cup \widetilde{D}$, $\tilde{C} \cup \widetilde{D} \subseteq X -(\widetilde{U} \cup \widetilde{V})$, $(\widetilde{U} \cup \widetilde{V}) \cap \tilde{C} \neq \widetilde{\emptyset}_I$ and $(\widetilde{U} \cup \widetilde{V}) \cap \widetilde{D} \neq \widetilde{\emptyset}_I$. Since $\widetilde{U}$ and $\widetilde{V}$ are intuitionistic semi * C_1- connected, $\widetilde{U} \cap \tilde{C} = \widetilde{\emptyset}_I$ or $\widetilde{U} \cap \widetilde{D} = \widetilde{\emptyset}_I$ and $\widetilde{V} \cap \tilde{C} = \widetilde{\emptyset}_I$ or $\widetilde{V} \cap \widetilde{D} = \widetilde{\emptyset}_I$. Since $\widetilde{U} \cap \widetilde{V} \neq \widetilde{\emptyset}_I$, $\tilde{p}_{IV} \in \widetilde{U} \cap \widetilde{V}$. **Case (i)** Let $\widetilde{U} \cap \tilde{C} = \widetilde{\emptyset}_I$ and $\widetilde{V} \cap \tilde{C} = \widetilde{\emptyset}_I$. Then $(\widetilde{U} \cap \tilde{C}) \cup (\widetilde{V} \cap \tilde{C}) = \widetilde{\emptyset}_I \Rightarrow (\widetilde{U} \cup \widetilde{V}) \cap \tilde{C} = \widetilde{\emptyset}_I$ which is a contradiction. **Case (ii)** Let $\widetilde{U} \cap \widetilde{D} = \widetilde{\emptyset}_I$ and $\widetilde{V} \cap \widetilde{D} = \widetilde{\emptyset}_I$. Then $(\widetilde{U} \cap \widetilde{D}) \cup (\widetilde{V} \cap \widetilde{D}) = \widetilde{\emptyset}_I \Rightarrow (\widetilde{U} \cup \widetilde{V}) \cap \widetilde{D} = \widetilde{\emptyset}_I$ which is a contradiction. **Case (iii)** Let $\widetilde{U} \cap \tilde{C} = \widetilde{\emptyset}_I$ and $\widetilde{V} \cap \widetilde{D} = \widetilde{\emptyset}_I$. Then $\tilde{p}_{IV} \notin \tilde{C}$ and $\tilde{p}_{IV} \notin \widetilde{D}$. This is impossible because $\tilde{p}_{IV} \in \widetilde{U} \cap \widetilde{V} \subseteq \tilde{C} \cup \widetilde{D}$. **Case (iv)** Let $\widetilde{U} \cap \widetilde{D} = \widetilde{\emptyset}_I$ and $\widetilde{V} \cap \tilde{C} = \widetilde{\emptyset}_I$. This case is similar to case (iii). Hence from the above four cases $\widetilde{U} \cup \widetilde{V}$ is an intuitionistic semi * C_1- connected.

Theorem 3.16. Let (X, τ) be an ITS and $\widetilde{U}$, $\widetilde{V}$ be any two IS of X. If $\widetilde{U}$, $\widetilde{V}$ are intuitionistic semi * C_2- connected and $\widetilde{U} \cap \widetilde{V} \neq \widetilde{\emptyset}_I$, then $\widetilde{U} \cup \widetilde{V}$ is also an intuitionistic semi * C_2- connected.

Xi'an Shiyou Daxue Xuebao (Ziran Kexue Ban)/
Journal of Xi'an Shiyou University, Natural Sciences Edition
ISSN: 1673-064X
E-Publication: Online Open Access
Vol: 66 Issue 05 | 2023
DOI 10.17605/OSF.IO/YWEC2

Proof. Let $\widetilde{U}$, $\widetilde{V}$ be intuitionistic semi * C_2- connected. Suppose $\widetilde{U} \cup \widetilde{V}$ is not an intuitionistic semi * C_2- connected. Then there exist an IS*O set $\tilde{C}$ and $\widetilde{D}$ such that $\widetilde{U} \cup \widetilde{V} \subseteq \tilde{C} \cup \widetilde{D}$,$(\widetilde{U} \cup \widetilde{V}) \cap \tilde{C} \cap \widetilde{D} = \widetilde{\emptyset}_I$,$(\widetilde{U} \cup \widetilde{V}) \cap \tilde{C} \neq \widetilde{\emptyset}_I$ and $(\widetilde{U} \cup \widetilde{V}) \cap \widetilde{D} \neq \widetilde{\emptyset}_I$. Since $\widetilde{U}$ and $\widetilde{V}$ are intuitionistic semi * C_2- connected, $\widetilde{U} \cap \tilde{C} = \widetilde{\emptyset}_I$ or $\widetilde{U} \cap \widetilde{D} = \widetilde{\emptyset}_I$ and $\widetilde{V} \cap \tilde{C} = \widetilde{\emptyset}_I$ or $\widetilde{V} \cap \widetilde{D} = \widetilde{\emptyset}_I$. Since $\widetilde{U} \cap \widetilde{V} \neq \widetilde{\emptyset}_I$, $\tilde{p}_{IV} \in \widetilde{U} \cap \widetilde{V}$. **Case (i)** Let $\widetilde{U} \cap \tilde{C} = \widetilde{\emptyset}_I$ and $\widetilde{V} \cap \tilde{C} = \widetilde{\emptyset}_I$. Then $(\widetilde{U} \cap \tilde{C}) \cup (\widetilde{V} \cap \tilde{C}) = \widetilde{\emptyset}_I \Rightarrow (\widetilde{U} \cup \widetilde{V}) \cap \tilde{C} = \widetilde{\emptyset}_I$ which is a contradiction. **Case (ii)** Let $\widetilde{U} \cap \widetilde{D} = \widetilde{\emptyset}_I$ and $\widetilde{V} \cap \widetilde{D} = \widetilde{\emptyset}_I$. Then $(\widetilde{U} \cap \widetilde{D}) \cup (\widetilde{V} \cap \widetilde{D}) = \widetilde{\emptyset}_I \Rightarrow (\widetilde{U} \cup \widetilde{V}) \cap \widetilde{D} = \widetilde{\emptyset}_I$ which is a contradiction. **Case (iii)** Let $\widetilde{U} \cap \tilde{C} = \widetilde{\emptyset}_I$ and $\widetilde{V} \cap \widetilde{D} = \widetilde{\emptyset}_I$. Then $\tilde{p}_{IV} \notin \tilde{C}$ and $\tilde{p}_{IV} \notin \widetilde{D}$. This is impossible because $\tilde{p}_{IV} \in \widetilde{U} \cap \widetilde{V} \subseteq \tilde{C} \cup \widetilde{D}$. **Case (iv)** Let $\widetilde{U} \cap \widetilde{D} = \widetilde{\emptyset}_I$ and $\widetilde{V} \cap \tilde{C} = \widetilde{\emptyset}_I$. This case is similar to case (iii). Hence from the above four cases $\widetilde{U} \cup \widetilde{V}$ is an intuitionistic semi * C_2- connected.

Theorem 3.17. Let (X, τ) be an ITS and $\widetilde{U}$, $\widetilde{V}$ be any two IS of X. If $\widetilde{U}$ and $\widetilde{V}$ are overlapping intuitionistic semi * C_3- connected, then $\widetilde{U} \cup \widetilde{V}$ is also an intuitionistic semi * C_3- connected.

Proof. Assume $\widetilde{U} \cup \widetilde{V}$ is not an intuitionistic semi * C_3- connected thats lead us to there exist and IS*O sets $\tilde{E}$ and $\tilde{F}$ such that $\widetilde{U} \cup \widetilde{V} \subseteq \tilde{E} \cup \tilde{F}$, $\tilde{E} \cap \tilde{F} \subseteq X - (\widetilde{U} \cup \widetilde{V})$, $\tilde{E} \nsubseteq X - (\widetilde{U} \cup \widetilde{V})$, $\tilde{F} \nsubseteq X - (\widetilde{U} \cup \widetilde{V})$. Since $\widetilde{U}$ and $\widetilde{V}$ are intuitionistic semi * C_3- connected, $\tilde{E} \subseteq X - \widetilde{U}$ or $\tilde{F} \subseteq X - \widetilde{U}$ and $\tilde{E} \subseteq X - \widetilde{V}$ or $\tilde{F} \subseteq X - \widetilde{V}$. Also by hypothesis $\widetilde{U}$ and $\widetilde{V}$ are overlapping, there is a point p, ($\tilde{p}_I \in \widetilde{U}$, $\tilde{p}_{IV} \in \widetilde{V}$) or there is a point q, ($\tilde{q}_I \in \widetilde{V}$, $\tilde{q}_{IV} \in \widetilde{U}$). **Case (i)** Let $\tilde{E} \subseteq X - \widetilde{U}$ and $\tilde{E} \subseteq X - \widetilde{V}$. Then $\tilde{E} \subseteq (X - \widetilde{U}) \cap (X - \widetilde{V}) = X - (\widetilde{U} \cup \widetilde{V})$ which is contradiction to $\tilde{E} \nsubseteq X - (\widetilde{U} \cup \widetilde{V})$. **Case (ii)** Let $\tilde{F} \subseteq X - \widetilde{U}$ and $\tilde{F} \subseteq X - \widetilde{V}$. This is similar to case (i). **Case (iii)** Let $\tilde{E} \subseteq X - \widetilde{U}$ and $\tilde{F} \subseteq X - \widetilde{V}$. Suppose there is a point p, ($\tilde{p}_I \in \widetilde{U}$, $\tilde{p}_{IV} \in \widetilde{V}$). Since $\tilde{E} \subseteq X - \widetilde{U}$ and $\tilde{F} \subseteq X - \widetilde{V}$, $\widetilde{U} \cup \widetilde{V} \subseteq \tilde{E} \cup \tilde{F} \subseteq (X - \widetilde{U}) \cup (X - \widetilde{V}) = X - (\widetilde{U} \cap \widetilde{V})$. Therefore $\widetilde{U} \cap \widetilde{V} \subseteq X - (\widetilde{U} \cup \widetilde{V}) = (X - \widetilde{U}) \cup (X - \widetilde{V})$. We have $\tilde{p}_I \in \widetilde{U}$ and $\tilde{p}_{IV} \in \widetilde{V} \Rightarrow \tilde{p}_{IV} \in \widetilde{U} \Rightarrow \tilde{p}_{IV} \in \widetilde{U} \cap \widetilde{V} \subseteq (X - \widetilde{U}) \cap (X - \widetilde{V}) \Rightarrow \tilde{p}_{IV} \in X - \widetilde{U}$ and $\tilde{p}_{IV} \in X - \widetilde{V}$ which is a contradiction. Similarly if there is a point q, ($\tilde{q}_I \in \widetilde{V}$, $\tilde{q}_{IV} \in \widetilde{U}$), we get a contradiction. **Case (iv)** Let $\tilde{E} \subseteq X - \widetilde{V}$ and $\tilde{F} \subseteq X - \widetilde{U}$. This is similar to case (iii). Therefore from the above four cases $\widetilde{U} \cup \widetilde{V}$ is an intuitionistic semi * C_3- connected.

Theorem 3.18. Let (X, τ) be an ITS and $\widetilde{U}$, $\widetilde{V}$ be any two IS of X. If $\widetilde{U}$ and $\widetilde{V}$ are overlapping intuitionistic semi * C_4- connected, then $\widetilde{U} \cup \widetilde{V}$ is also an intuitionistic semi * C_4- connected.

Proof. The proof is similar to previous theorem.

4. INTUITIONISTIC SEMI * COMPACT SPACES

Definition 4.1. Let $\widetilde{Đ}$ be a family of IS*O sets of X, and let (X, τ) be an ITS. Then the collection $\widetilde{Đ}$ is called an intuitionistic semi * open cover (summarizing IS*-OC) of X if $\cup \widetilde{Đ} = \tilde{X}_I$.

Xi'an Shiyou Daxue Xuebao (Ziran Kexue Ban)/
Journal of Xi'an Shiyou University, Natural Sciences Edition
ISSN: 1673-064X
E-Publication: Online Open Access
Vol: 66 Issue 05 | 2023
DOI 10.17605/OSF.IO/YWEC2

Definition 4.2. An ITS (X, τ) is said to be an intuitionistic semi * compact (summarizing IS*-cpt) if every IS*-OC of X has a finite sub cover.

Theorem 4.3. Let (X, τ) be an ITS. Then the following results hold.

(i) Every IS*-cpt implies intuitionistic compact.

(ii) Every intuitionistic semi compact implies IS*-cpt.

Proof. (i) Let (X, τ) be an IS*-cpt and $\{\widetilde{U}_\alpha\}$ be an intuitionistic open cover for X. Then $\{\widetilde{U}_\alpha\}$ is an IS*-OC for X. Since X is an IS*-cpt, $\{\widetilde{U}_\alpha\}$ has a finite subcover. Hence X is an intuitionistic compact. (ii) Let (X, τ) be an intuitionistic semi compact and $\{\widetilde{D}_\alpha\}$ be an IS*-OC for X. Then $\{\widetilde{D}_\alpha\}$ is an intuitionistic semi open cover for X. Since X is an intuitionistic semi compact, $\{\widetilde{D}_\alpha\}$ has a finite subcover. Hence (X, τ) is an IS*-cpt.

Theorem 4.4. Let (X, τ) be an ITS. Then (X, τ) is IS*-cpt if and only if every family of IS*C sets in X with void intersection has a finite subfamily with void intersection.

Proof. Let (X, τ) be an IS*-cpt and $\{\widetilde{U}_\alpha\}_{\alpha\in J}$ be a family of IS*C sets in X such that $\cap\{\widetilde{U}_\alpha\}_{\alpha\in J} = \widetilde{\emptyset}_I$. Then $\cup\{X - \widetilde{U}_\alpha\}_{\alpha\in J} = \tilde{X}_I$ is an IS*-OC for X. Since X is an IS*-cpt, X has a finite subcover, namely $\{X - \widetilde{U}_{\alpha 1}, X - \widetilde{U}_{\alpha 2}, ..., X - \widetilde{U}_{\alpha n}\}$ for X. Therefore $\tilde{X} = \cup_{i=1 \text{ to } n}\{X - \widetilde{U}_{\alpha i}\}$. Thus $\cap_{i=1 \text{ to } n}\{\widetilde{U}_{\alpha i}\} = \widetilde{\emptyset}_I$. Conversely, assume that every family of IS*C sets in (X, τ) with empty intersection has a finite subfamily with void intersection. Let $\{\widetilde{D}_\alpha\}_{\alpha\in J}$ be an IS*-OC for (X, τ) . Then $\cup\{\widetilde{D}_\alpha\}_{\alpha\in J} = \tilde{X}_I$. Therefore $\{X - \widetilde{D}_\alpha\}_{\alpha\in J} = \widetilde{\emptyset}_I$. Since $X - \widetilde{D}_\alpha$ is IS*C set for each α ∈ J, by hypothesis there is a finite subfamily has a empty intersection. That is $\cap_{i=1 \text{ to } n}(X - \widetilde{D}_\alpha) = \widetilde{\emptyset}_I$. Then $\cup_{i=1 \text{ to } n}\widetilde{D}_\alpha = \tilde{X}_I$. Hence (X, τ) is an IS*-cpt.

Theorem 4.5. Let (X, τ_1) and (Y, τ_2) be any two ITS and $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ be an IS*O function. If (Y, τ_2) is an IS*-cpt, then (X, τ_1) is an IS*-cpt.

Proof. Let $\{\tilde{F}_\alpha\}$ be an IS*-OC for (X, τ_1). Then $\{f(\tilde{F}_\alpha)\}$ is an IS*-OC for (Y, τ_2). Since (Y, τ_2) is an IS*-cpt, $\{f(\tilde{F}_\alpha)\}$ has an finite subcover, namely $\{f(\tilde{F}_{\alpha 1}), f(\tilde{F}_{\alpha 2}), ..., f(\tilde{F}_{\alpha n})\}$. Therefore $\{\tilde{F}_{\alpha 1}, \tilde{F}_{\alpha 2}, ..., \tilde{F}_{\alpha n}\}$ is a finite subcover for(X, τ_1). Hence (X, τ_1) is an IS*-cpt.

Theorem 4.6. Let (X, τ_1) and (Y, τ_2) be any two ITS and $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ be an IS*O function. If (Y, τ_2) is an IS*-cpt, then (X, τ_1) is an intuitionistic compact.

Proof. Let $\{\tilde{E}_\alpha\}$be an intuitionistic open cover for (X, τ_1). Since f is an IS*O and $\{\tilde{E}_\alpha\}$ is an intuitionistic open cover for (Y, τ_2), $\{f(\tilde{E}_\alpha)\}$ is an IS*-OC for (Y, τ_2). Since (Y, τ_2) is an IS*-compact, $\{f(\tilde{E}_\alpha)\}$ has an finite subcover,namely $\{f(\tilde{E}_{\alpha 1}), f(\tilde{E}_{\alpha 2}), ..., f(\tilde{E}_{\alpha n})\}$. Therefore $\{\tilde{E}_{\alpha 1}, \tilde{E}_{\alpha 2}, ..., \tilde{E}_{\alpha n}\}$ is a finite subcover for(X, τ_1). Hence (X, τ_1) is an intuitionistic compact.

Theorem 4.7. Let (X, τ_1) and (Y, τ_2) be any two ITS and $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ be a surjection and IS*-Cts function. If (X, τ_1) is an IS*-cpt, then (Y, τ_2) is an intuitionistic compact.

Proof. Let $\{\tilde{F}_\alpha\}$ be an intuitionistic open cover for (Y, τ_2). Since f is an IS*-Cts, $\{f^{-1}(\tilde{F}_\alpha)\}$ is an IS*-OC for (X, τ_1). Since (X, τ_1) is an IS*-cpt, $\{f^{-1}(\tilde{F}_\alpha)\}$ has finite subcover, namely

ISSN: 1673-064X
E-Publication: Online Open Access
Vol: 66 Issue 05 | 2023
DOI 10.17605/OSF.IO/YWEC2

$\{f^{-1}$ ($\tilde{F}_{\alpha 1}$), f^{-1} ($\tilde{F}_{\alpha 2}$), ..., f^{-1}($\tilde{F}_{\alpha n}$)}. Therefore {$\tilde{F}_{\alpha 1}$, $\tilde{F}_{\alpha 2}$, ..., $\tilde{F}_{\alpha n}$ } is a finite subcover for (Y, τ_2). Hence (Y, τ_2) is an intuitionistic compact.

Definition 4.8. An ITS (X, τ) is said to be an intuitionistic semi * Lindelof (summarizing IS*-L) if every IS*-OC contains countable subcover.

Theorem 4.9. Let f : (X, τ_1) → (Y, τ_2) be an surjection, IS*-Cts and (X, τ_1) be an IS*-L. Then (Y, τ_2) is an intuitionistic lindelof.

Proof. Let (X, τ_1) be an IS*-L and {$\tilde{F}_\alpha$ } be an intuitionistic open cover for (Y, τ_2).Then {f^{-1} ($\tilde{F}_\alpha$)} is an IS*-OC for (X, τ_1). Since (X, τ_1) is IS*-L, {f^{-1} ($\tilde{F}_\alpha$)} contains a countable subcover say, {f^{-1} ($\tilde{F}_{\alpha n}$)}. Then { $\tilde{F}_{\alpha n}$} has a countable subcover for (Y, τ_2). Thus (Y, τ_2) is an intuitionistic lindelof.

Theorem 4.10. Let f : (X, τ_1) → (Y, τ_2) be an surjection, IS*-Irresolute and (X, τ_1) be an IS*-L. Then (Y, τ_2) is an IS*-L.

Proof. Let (X, τ_1) be an IS*-L and {$\tilde{F}_\alpha$} be an IS*-OC for (Y, τ_2). Then {f^{-1} ($\tilde{F}_\alpha$)} is an IS*-OC for (X, τ_1). Since (X, τ_1) is IS*-L, {f^{-1} ($\tilde{F}_\alpha$)} contains a countable subcover say, {f^{-1} ($\tilde{F}_{\alpha n}$)}. Then {$\tilde{F}_{\alpha n}$}is a countable subcover for (Y, τ_2). Thus (Y, τ_2) is an IS*-L.

Theorem 4.11. Let f : (X, τ_1) → (Y, τ_2) be an intuitionistic pre semi * open and (Y, τ_2) be an IS*-L. Then (X, τ_1) is an IS*-L.

Proof. Let (Y, τ_2) be an IS $*$−L and {$\widetilde{D}_\alpha$} be an IS*-OC for (X, τ_1). Then {f($\widetilde{D}_\alpha$)} is an IS*-OC for Y . Since (Y, τ_2) is IS*-L, {f($\widetilde{D}_\alpha$)} contains a countable subcover say, {f($\widetilde{D}_{\alpha n}$)}. Then {$\widetilde{D}_{\alpha n}$} is a countable subcover for (X, τ_1). Thus (X, τ_1) is an IS*-L.

Theorem 4.12. Let f : (X, τ_1) → (Y, τ_2) be an IS*O function and (Y, τ_2) be an IS*-L. Then (X, τ_1) is an intuitionistic lindelof.

Proof. Let (Y, τ_2) be an IS $*$−L and {$\widetilde{D}_\alpha$} be an intuitionistic open cover for (X, τ_1). Then {f($\widetilde{D}_\alpha$)} is an IS*-OC for (Y, τ_2). Since (Y, τ_2) is IS*-L, {f($\widetilde{D}_\alpha$)} contains a countable subcover say, {f($\widetilde{D}_{\alpha n}$)}. Then {f($\widetilde{D}_{\alpha n}$)} is a countable subcover for (X, τ_1). Thus (X, τ_1) is an intuitionistic lindelof.

5. CONCLUSION

The different qualities of intuitionistic semi * connectedness and compactness are covered in this article. We will continue to investigate different concepts, such as maximal and minimal open sets, separation axioms in IS*O sets.

6. ACKNOWLEDGEMENT

My heartful thanks to my guide Dr. M. Navaneethakrishnan for his valuable guidance on writing this paper and thanks to the referees for their time and comments.

ISSN: 1673-064X
E-Publication: Online Open Access
Vol: 66 Issue 05 | 2023
DOI 10.17605/OSF.IO/YWEC2

References

1. D. Coker, An introduction to intuitionistic topological space preliminary report , Akdeniz University, Turkey, 199
2. D. Coker, a Note on intuitionistic sets and intuitionistic points, Turk. J. Math, 20(3), 1996, 343 - 351.
3. D. Coker, An introduction to intuitionistic topological spaces, Busefal, 81, 2000, 51 - 56.
4. D.Coker, Selma Ozcag on Connectedness in intuitionistic fuzzy special topological spaces, International Journal of Math. And Math.Science, 21(1), (1998), 33-40.
5. GirijaS. Selvanayki S., Ilango Gnanambal. "Semi connected spaces in intuitionistic topological spaces", Malaya Journal of Matematik, 2020
6. K.T.Atanassov, VII ITKKR's Session, June 1983 (Soa) (V. Sgurev, ed.), Central Sci. and Techn. Library, Bulg. Academy of Scienes, 1984.
7. Rathinakani, G. Esther, and M. Navaneethakrishnan, "A NEW CLOSURE OPERATOR IN INTUITIONISTIC TOPOLOGICAL SPACES."
8. Rathinakani, G. Esther, and M. Navaneethakrishnan, "A Study on Intuitionistic Semi * Open Set." Design Engineering (2021): 5043-5049.
9. Rathinakani, G. Esther, and M. Navaneethakrishnan, "On Intuitionistic Semi * Continuous Functions." Ratio Mathematica 44 (2022): 65.
10. Rathinakani, G. Esther, and M. Navaneethakrishnan, “Some New Operators on Intuitionistic Semi * Open Set.” Proceedings of ICCSMM-2022, Tiruchendur, Tamilnadu, India.
11. Sema Ozcag and Dogan Coker, a Note on Connectedness in Intuitionistic Fuzzy Special Topological Spaces, Internat. J. Math. And Math. Sci. Vol.23, No.1 (2000) 45-54.
12. Younis J. Yaseen and Asmaa G. Raouf (2009) “On generalization closed set and generalized continuity in intuitionistic topological sapce" University of Tirkit- College of Computer Science and Mathematics.