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ANALYTICAL AND NUMERICAL ANALYSES OF THE DAMAGE IN REPAIRED PLATES WITH DOUBLE COMPOSITE PATCH AND STIFFENERS UNDER TENSILE LOAD

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Abstract

This paper examines the probabilistic aspects of fracture mechanics for repaired cracks under tensile loading. Our approach utilizes a three-dimensional finite element method to evaluate the J-integral in plates that have been repaired and reinforced with bonded composite double patches and stiffeners. We analyzed the influence of crack length, patch thickness (ep), and the patch's mechanical properties on the J-integral. The numerical results align well with analytical solutions available in the literature. The methodology includes performing elastic assessments to predict the J-integral, employing statistical models to account for uncertainties in loads and material properties, and using established computational techniques based on structural reliability theory for probabilistic analysis. Monte Carlo simulations were conducted to characterize the probabilistic behavior of the J-integral. The results indicate that repaired plates exhibit greater durability than those left unrepaired.

Index Terms: Analytical Solution, Failure, Safety, Composite, Repaired Plate and Stiffener, Probabilistic Analysis.

1. INTRODUCTION

Predicting different failure scenarios in aeronautical structures is important for their optimal design. Probabilistic models have been developed to estimate statistics for various failure scenarios, providing a means to assess the reliability of these structures. Today, probabilistic fracture mechanics is widely applied across several industries, including oil and gas, nuclear, marine, aerospace, and others [1,2].Bonded composite repair is an effective and cost-efficient technique in which composite patches are applied to one or both sides of a cracked component to extend its fatigue life [3]. This repair method shows great potential and has already been adopted in various sectors, such as aeronautics, marine structures, the oil and gas industry, and civil engineering. Predicting the service life of structures under tensile loads is essential for effective design, enabling planned inspections and maintenance.

Composite patch repairs offer several advantages over mechanically fastened doublers, including improved fatigue resistance, reduced corrosion risk, and adaptability to complex aerodynamic shapes [4-9]. Evaluating the J-integral for cracked structures typically involves numerical analysis and engineering estimation methods. Historically, elastic finite element methods (FEM) have been widely used for numerical investigations [10-13]. Many researchers have applied FEM to study damaged aerospace materials and structures, highlighting the importance of J-integral evaluation for failure prediction and structural integrity. Accurate calculation of the J-integral is vital for assessing structural reliability, which has led to the development of various techniques to obtain this parameter [14, 15].

This paper, presents a probabilistic fracture-mechanics of repaired cracks under tensile loading. A three-dimensional finite element method is used to evaluate the J integral around repaired cracks in plates with bonded composite double patch and stiffeners. The effect of the crack length, the patch thickness (ep) and the mechanical properties of the patch on The J- integral variation were studied. Monte Carlo simulation was used to determine the probabilistic characteristics of the J-integral of repaired crack. This method was also used to predict the failure probability based on crack growth.

2. GEOMETRICAL MODEL

The basic geometry of the cracked structure considered in this study is shown in Figure.1. Consider a plate with the following dimensions: height Hp=254 mm, width wp=254 mm, thickness ep=3 mm. The plate is subjected to uniaxial tensile load giving a remote stress state of σ=100 MPa for elastic analysis. A central crack of length 2a perpendicular to the loading axis is supposed to exist in the plate. This crack is repaired with unidirectional Boron/Epoxy composites patches. The ply orientation is parallel to the loading axis. The initial dimensions of the patch are: height Hr=80 mm, width wr=100 mm and thickness er=2 mm. The adhesive is used to bond the patch on cracked plate: FM 73, Epoxy adhesive. The adhesive thickness (ea) is taken equal to 0.2 mm.

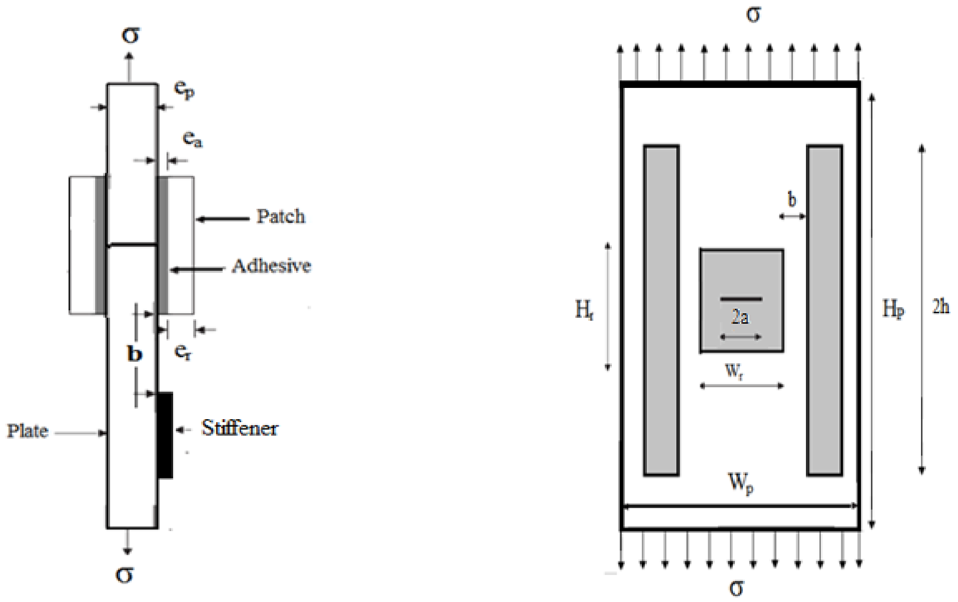

Figure 1: Geometrical model

The elastic properties of the plate, the patch and the adhesives are given in Table1 [15]

Table1

	Aluminum 2024-T3	Boron/epoxy	Graphite/epoxy	Glass/epoxy	Adhesive FM-73
E1(GPa)	72	200	150	127.5	2.55
E2(GPa		19.6	25	9	
E3(GPa)		19.6	2.5	4.8	
υ12	0.3	0.3	0.21	0.342	0.32
υ13		0.28	0.21	0.342	
υ23		0.28	0.21	0.38	
G12(GPa)		7.2	7.2	4.8	
G13(GPa)		5.5	5.5	4.8	
G23(GPa)		5.5	5.5	2.55	

3. FINITE ELEMENT MODELING

Figure.2 presents the three-dimensional numerical model developed for this study which consists of a cracked Al 2024 T3 plate repaired by composite patch and stiffener. The structure has been meshed globally using elements of the type C3D8. (An 8-node linear brick) were used to build a quarter model of the structure. Values of the J-integral were extracted using a domain integral method implemented within ABAQUS. In this study, the entire mesh of the model was built from two blocks consisting of the structures block and the crack's block. A series of tests were undertaken to estimate mesh sensitivity on the results of the J-integral. An initial mesh of 10568 elements in total was employed and refined several times (17811, 25736, 34058) until reaching 41353 elements.

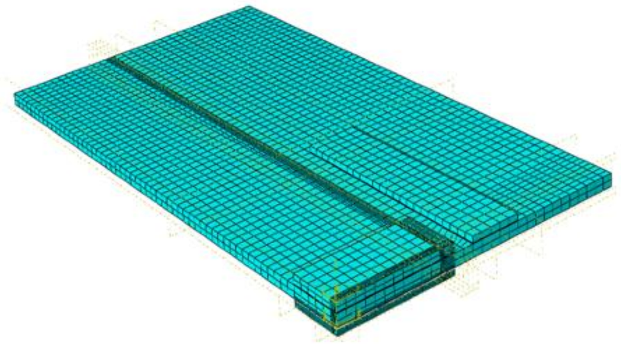

Figure 2: Typical Mesh Model of the Quarter of the Structure

4. RESULTS AND DESCUSSIONS

4.1 Analytical formulation

The parameter J is defined from the following contour integral

$$J = \int_{r} \left(w\,d\,y - T_i \frac{\partial u_i}{\partial x} d\,s \right) \tag{1}$$

Where Γ is a contour of integration surrounding the tip of the crack, as shown in Figure (3), ds the length element on Γ, Ti and ui being the components of the stress vector and displacement vector at a point on Γ. The strain energy density w is defined as

$$w = \int_0^{c_{ij}} \sigma_{i\,j}\, d\varepsilon_{i\,j} \tag{2}$$

Where σij and εij are the components of stress and strain tensors at the current point on the contour Γ.

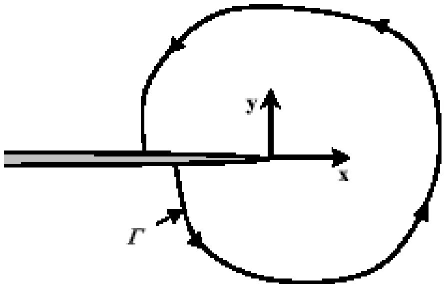

Figure 3: Integration Contour [16]

Introduced by G.R. Irwin [17], stress intensity factors correspond to specific kinematics of crack movement. In the context of linear fracture mechanics, stresses and strains in the vicinity of a crack admit an asymptotic expansion with a singular term written as. Stress fields are expressed using stress intensity factors.

$$\begin{cases} \sigma_{xx} = \dfrac{K_1}{\sqrt{2\pi r}} \cos\dfrac{\theta}{2}\left(1-\sin\dfrac{\theta}{2}\sin\dfrac{3\theta}{2}\right) \\ \sigma_{yy} = \dfrac{K_1}{\sqrt{2\pi r}} \cos\dfrac{\theta}{2}\left(1+\sin\dfrac{\theta}{2}\sin\dfrac{3\theta}{2}\right) \\ \tau_{xy} = \dfrac{K_1}{\sqrt{2\pi r}} \cos\dfrac{\theta}{2}\sin\dfrac{\theta}{2}\cos\dfrac{3\theta}{2} \end{cases} \tag{3}$$

Figure. 4 shows a comparative analysis of the J-integral variation obtained from the FEM results developed in this work, and that resulting from an analytical solution (eq. 5). The stress intensity factor (SIF) is the only significant parameter, the relation between the far applied stress on the plat (σ) and the stress Intensity factor (KI) is as follows[17,18]

$$K_I = Y\sigma\sqrt{\pi a} \tag{4}$$

The elastic of the J-integral for mode I (Je) can be found

$$J_e = \frac{K_I^2}{E'} \tag{5}$$

Where E’=E for plane stress and   E’=E/ (1-υ2) for plane strain It can be seen, according to Fig. 4 that the presence of the patch has a considerable effect on the J-integral variation at the crack tip. It shows that the patch repair highly decreases the J-integral, the maximum reduction of J-integral is about to 75% of the crack length a   35mm. which are confirmed by the distribution of Von Mises stresses presented in Figure 5. The analytical solution gives a good agreement of the J- integral compared with the finite element method (see figure 4).

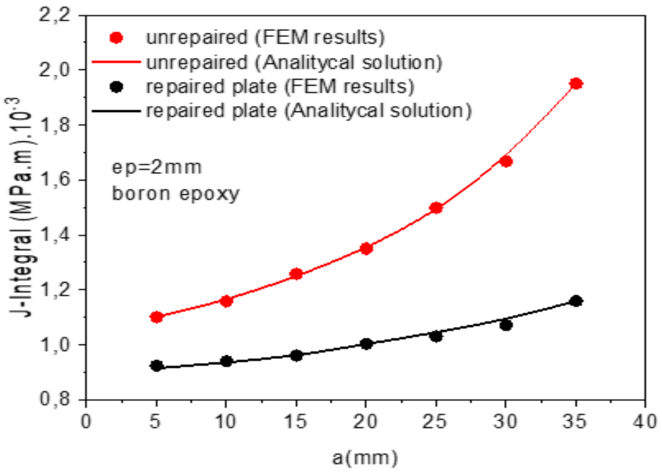

Figure 4: Variation of the J-integral according to the crack length for the cases with and without double patch and stiffener for analytical and FEM solution

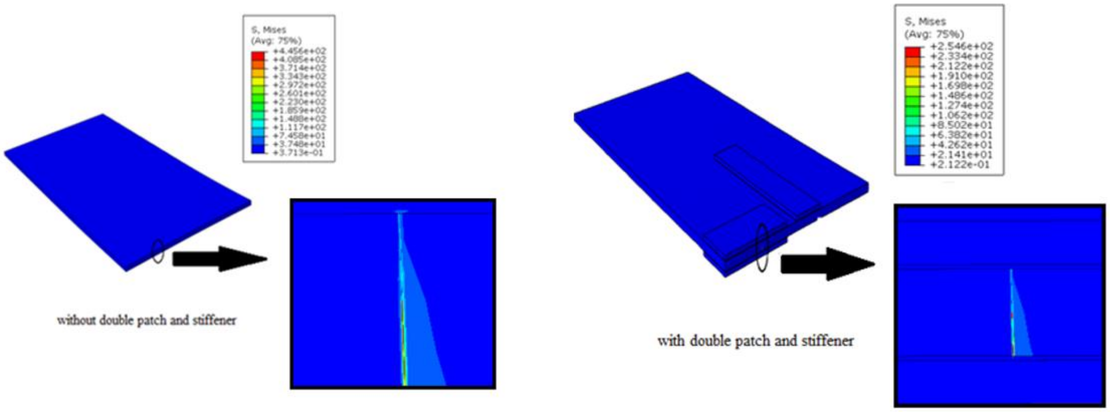

Figure 5: Maximum stress analysis with and without double patch and stiffener

4.2. Comparison between single patch and double patched with stiffener

To better illustrate the beneficial effect of the double symmetric patch, Figure 6 presents the variation of the asymptotic J-integral as a function of crack length for both single and double patches with a stiffener. In both cases, the J-integral increases asymptotically with crack length. The presence of the double patch and stiffener significantly influences the J-integral variation at the crack tip. As shown in Figure 7, the maximum relative variation in the J-integral reaches approximately 20% for a crack length of a=35mm.

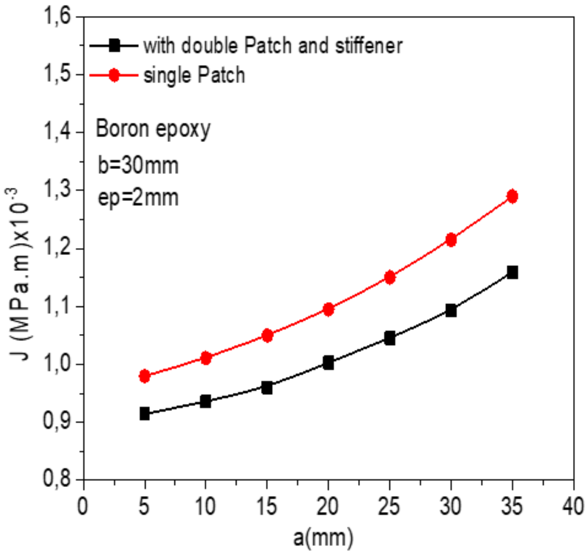

Figure 6: Variation of the asymptotic J- integral according to the crack length for a single and double patch with stiffener

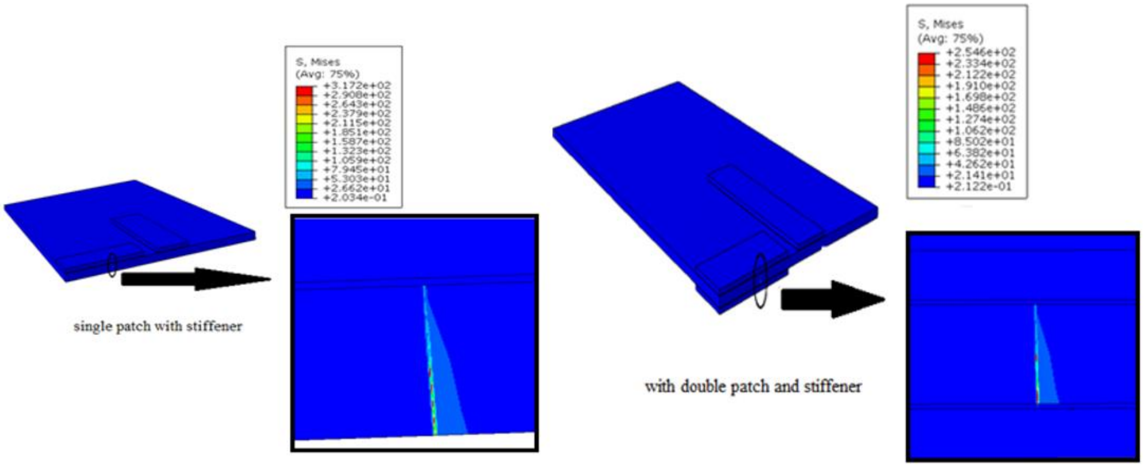

Figure 7: Maximum stress analysis with single and double patch and stiffener

4.3. Effect of the patch thickness

Figure 8 shows the variation of the J-integral as a function of crack length for different patch thicknesses. It is observed that, underpurely mechanical loading, increasing the patch thickness by 50% results in a proportional decrease in the J-integral at the crack tip. Clearly, the J-integral is inversely proportional to the patch thickness. However, the J-integral reduction becomes less significant when the patch thickness exceeds 2 mm. Specifically, the J-integral decreases by approximately 13% for a 2 mm thick patch compared to a 1 mm thick patch (see Figure 9). Optimizing these parameters enhances the efficiency of structural repairs.

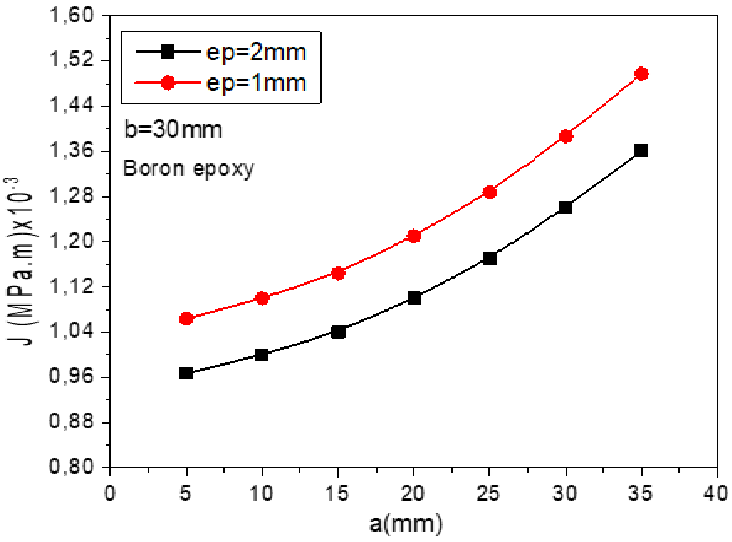

Figure 8: Variation of the J-integral according to the patch thickness for different cracks

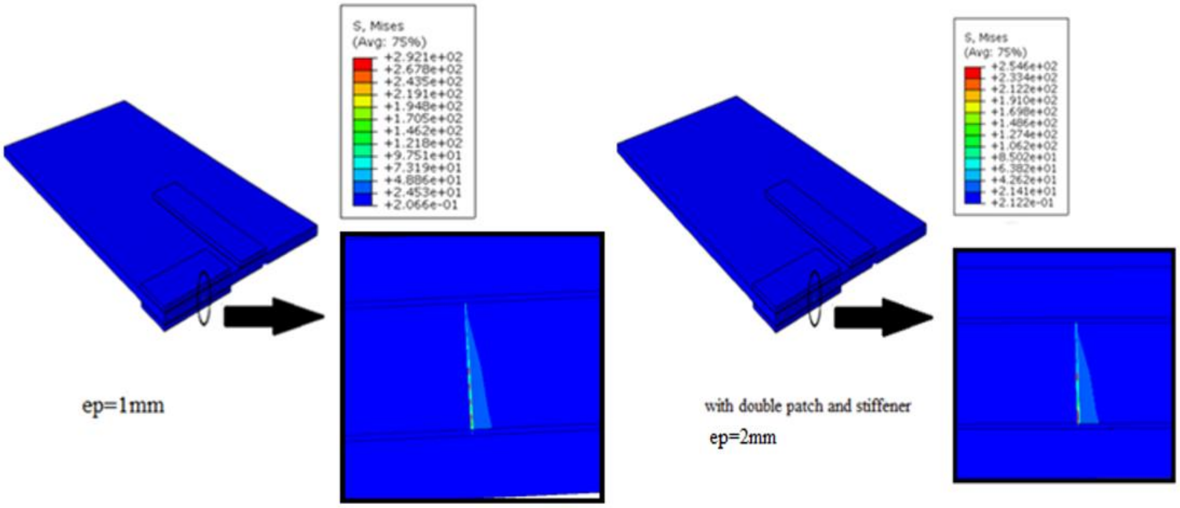

Figure 9: Maximum stress analysis for two different thickness of the patch

4.4. Effect of the patch properties

Figure 10 illustrates the variation of the J-integral with respect to crack length for three different types of patches (Boron/epoxy and Glass/epoxy). The results indicate that the value of the J-integral increases as the crack length grows. Among the tested materials, the Boron/epoxy patch demonstrates superior performance in the repair of damaged structures. The properties of the patch play a crucial role in the effectiveness of the repair process using bonded composite patches, as highlighted in Figures 10 and 11.

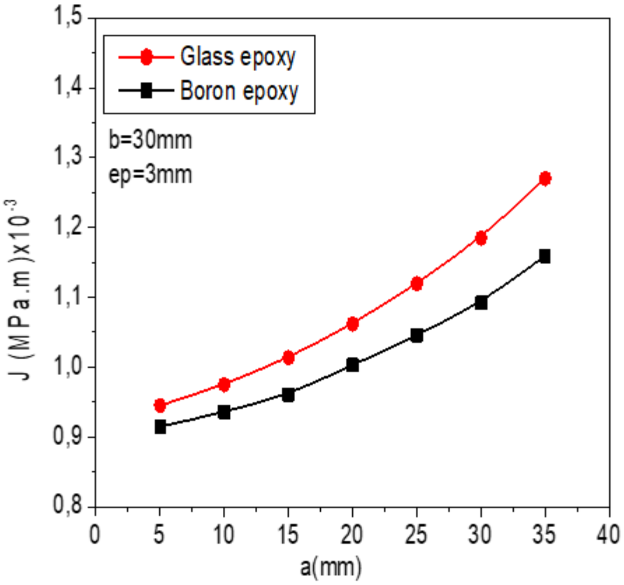

Figure 10: Variation of the J-integral according to the cracks for two different properties of the patch

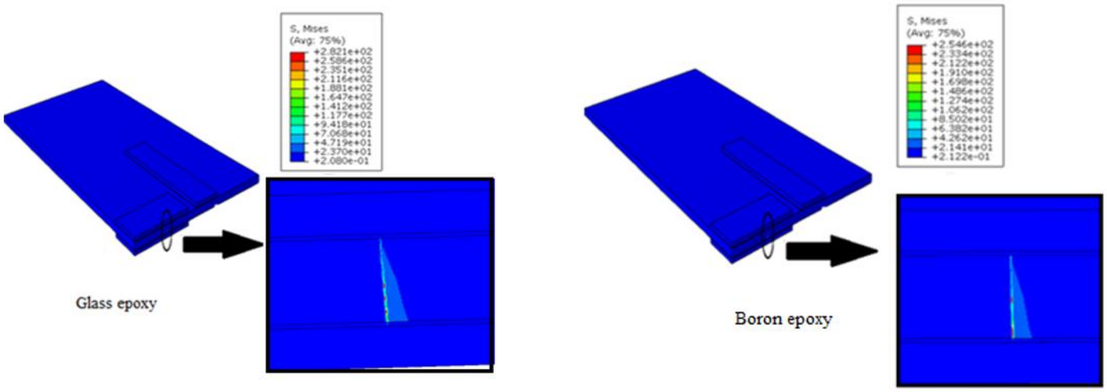

Figure 11: Maximum stress analysis for two different properties of the patch (Boron/ epoxy and Glass/ epoxy)

5. NUMERICAL PREDICTION

The random components are: geometric parameters, H, W, and L, material tensile parameters, E,υ, length of cracks (a) varies from 5 to 35mm.The safety margin (J) (xi) is the probabilistic design rule, which defines the plate safety by the condition (J) (xi) >0 and the plate failure by (J) (xi) ≤ 0. The probability density functions of (J) are calculated with a FORTRAN program using the Monte Carlo method. To guarantee a high level of accuracy in our results, we ran 10^5 simulations.

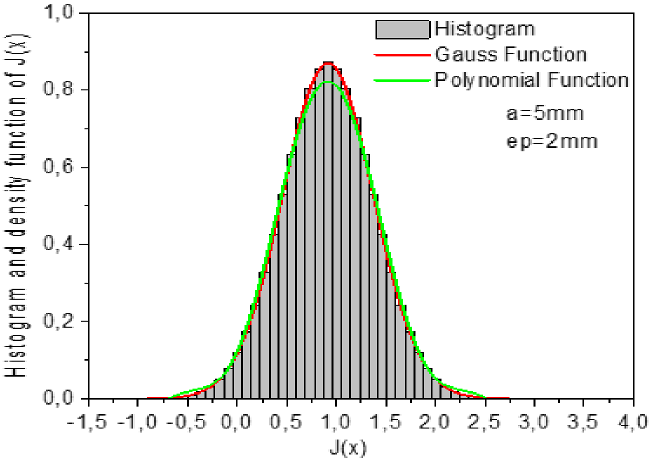

Figure 12: Histogram and Pobability density function of J (x)

Figure 12 presents the histogram of the J-integral function obtained through Monte Carlo simulations. The probability density function (pdf) is derived by fitting the histogram with theoretical models. Two distribution models were evaluated: the Gaussian distribution and a ninth-order polynomial. The Gaussian distribution offers a satisfactory approximation of the J-integral's probability density, providing a reliable estimate of the mean. Figures 13 display the probability density of J(x) for plates repaired with double patches and stiffeners, compared to unrepaired plates. The results reveal that the margin increases significantly with the uncertainties associated with the absence of patches and stiffeners, leading to a higher probability of failure. The uncertainty in the unrepaired plate has a notable impact on increasing the failure probability.

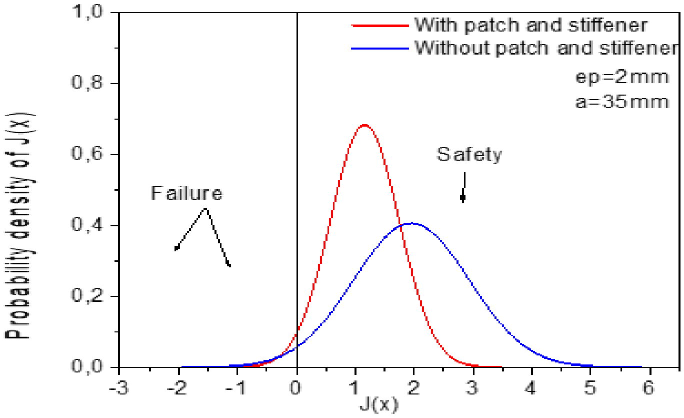

Figure 13: Pobability density function of J (x) with and without double patch and stiffener

6. CONCLUSION

The results presented in this study show that composite patch repair of a cracked structure considerably reduces the integral J around the crack face, resulting in an increase in the fatigue life of the structure. This increase is all the more significant when a symmetrical double patch is used. The presence of a stiffener also reduces stresses around the crack front, contributing to improved fatigue life. The numerical results are in good agreement compared with the analytical solution found in the literature .The probabilistic analysis carried out showed that the Gaussian distribution gives a good approximation to the probability density of the J integral function. In addition, same methods were used later to predict the failure probability based on the crack growth. The uncertainty in the unrepaired plate has a significant effect on increasing the probability of failure.

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