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REVISITING OPTIMIZATION TECHNIQUES FOR INCREASED EFFECTIVENESS

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Abstract

Traditional optimization is critical, using differential calculus to find the optimal value for unconstrained and constrained objective functions. Use differential calculus-based methods to achieve an optimum resolution of requests involving constant and differential tasks and adequate circumstances for finding an optimal reaction for emotional and restrained, single and multi-variable optimization problems with equivalence and in equivalence restraints. Make a distinction between local, global, and extreme inflection points. The purpose is to use the direct substitution method, Language multipliers method, and Kuhn-Tucker method have also been discussed to realize the optimal quantity of an empirical act with equality and inequality constraints. The findings are traditional optimization approaches achieve an optimal result of specific difficulties that involve continuous and differentiable functions. These procedures are logical, along with getting into the advantage of differential calculus to find maxima and minima points for a constrained single and multiple variable continuous function .The originality of my paper is using various methods for comparing Lagrange method, Kuhn-Tucker condition.

Keywords: Classical Optimization Technique, Closed Interval, Convex Function, Global Least, LPP/NLPP, Optimization Tools, Regional Highest, Stationary Point.

1. INTRODUCTION

Optimality circumstances perform an essential part in the optimization of both academic and practical perspectives. First-order and second-order optimality has remained intensively advanced, e.g., conclusions closely linked to this paper in, [4,5,7,9,10,11,12,15,16,20,24] and the mentions. Relatively rare charities to greater-direct optimality are easy to use in the works, although they offer valuable information. The Non-linearity now is greater. For collection-estimated optimization, we can refer to numerous newspapers [14, 18, 19]. Not long ago, by utilizing the higher-order contingent, asymptotic, and Studniarski derivatives, greater-sort Karush-Kuhn-Tucker laws used for regional feeble and strong results of setting-estimated troubles along with additional restrictions have been estimated in [17]. Used for the single cycle, they include greater-charge relationship negligence linked along with the actual charts. An extra improvement of that article is the usage of H¨ older measured sub regularity, an actual nonlinear measured predictability situation.

However, the outcomes are divided into two separate categories the multiplier instructions of H¨ older metric sub regularity just keep up with first-order relationship negligence and the ones with more outstanding orders sloppiness require the standard straight, measured sub regularity to be enforced. This great weakness motivates us to be for outcomes together with the more than good quality characteristics. In this paper, we achieve this objective by recommending and utilizing a new-found theory of pseudo-depending on derivatives and identifying appropriate analytical instruction. At this moment, it is better to define these two approaches for achieving our objective quickly. First, several available byproducts have been established with product presentations. The group derived [1], the Studniarski derived [23], the conditional epi derivative [11], and the variation setting [14] are thoroughly connected to the one we suggest and use in this paper. The person who reads is advised to read the comprehensive books [1,12,21,22] for extensive derivatives. However, these recognized products cannot be utilized to attain our objective mentioned above. Next, some greater-order essential condition offers info for inspecting an applicant for an optimal result when it previously gratifies all the circumstances of the smaller orders critically. So, the more accurate critical instructions are clear, the better important events are attained. In the current greater-order laws, non-adequate consideration is given to crucial instructions, and only relationship sloppiness of instruction one was concerned in optimality circumstances.

Considering the items in our understanding, we select a question set of common setting-esteemed questions to coordinate with current influences in the narrative. For the path optimality concepts, we balance standard ones: weak, Henig-appropriate, appropriate, and Brewin-appropriate results. The explanation is that a feeble result is prevalent, and categories of right solutions are essential. Specifically, to eradicate inconsistent results in enormous sets of ineffective and Pareto results, several concepts of appropriate productivity have been introduced, for instance, those in the sense of Benson [2], Borwein [3], Geoffrion [6], Henig [8], or of constructive appropriateness (see, e.g., [13]), etc. We notice that nearly all the available greater-order optimality circumstances are for feeble results, and extremely few possibilities have been achieved for these relevant results. Also, in several articles, for example [14,18,19,20,25,26], the dual forms of greater-order circumstances are kinds of the Fritz John multiplier law. At the same time, the Karush-Kuhn-Tucker single is further beneficial because it comprises the empirical act clearly, eliminating the circumstance that the rules are just some disproportionate assets of the restraints.

The standard optimization techniques are employed to acquire an optimal result for questions that incorporate constant and differentiable purposes. These approaches are critical and use differential calculus to find maxima and minima points for both unrestrained and restrained continuous goal purposes. This chapter will discuss the

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mandatory and adequate requirements for achieving an optimal result of Unrestrained individual and multiple variable optimization questions and constrained multi-variable optimization questions with fairness and inequality restraints.

The optimization Method is highly critical in real-life questions, study, and manufacturing functions. Optimization means to discover the most significant potential results out of the available substitutions under the given situation. The optimization method has remained successful in various areas. These multiple areas are Mechanism, Commercial, and Investment, Electric and Civil Engineering, Operation Research, Power Planning, Geophysical science, Molecular Developing [27]. From all these various areas, electric production is a highly critical area. Numerous questions arise in electrical energy structure. By utilizing the optimization method, we can solve all the questions in the power system. In the optimization method, several techniques differ on the kind of question. The paper will examine the standard optimization methods and essential and appropriate requirements for acquiring the optimal result of unrestrained individual and multi-variable optimization difficulties. Restrained multi-variable questions with fairness and inequality restrictions have been considered in part with illustrations. The traditional optimization methods are beneficial to achieve the optimum result of difficulties requiring constant and differentiable purposes. Such methods are critical to achieving the highest and lowest goals for unrestrained and restrained continuous goal functions. We use Lagrange's multiplier technique for fairness restrained questions and for inequality restrained, the Kuhn-Tucker requirements, for making optimal results.

2. UNCONSTRAINED OPTIMIZATION

2.1 Optimizing Single–Variable Functions

Figure 1 depicts the graph of a continuous function$y = f(x)$ of the single independent variable, x in the field (a, b). The domains the range of ideals of x. The domain limits (or endpoints) are generally called stationary (or critical) points. There are two categories of stationary points: inflection points and extreme points. The extreme points are either regional (or relative) or global (or absolute) extreme. Regional highest goals represent the highest or least standards of the task in the variety of ideals of the variable. In Fig. 1, points a,$x_{1,} x_{2,} x_{3,} x_{4,} x_{5,}$and b are all extreme off(x). The classical approach to the maxima and minima theory does not provide a direct method of obtaining the global (or absolute) highest (or lowest) amount. It only provides the method for determining the local (or relative) highest and lowest ideals. Mathematically, a utility y = f (x) is said to achieve its highest value at a point, x =x_0 , iff (x_0 + h) – f (x_0) < 0 or f (x_0+ h) <f (x_0) where h is a sufficiently small number about the moment x =x_0 . In other words, the point $x_0$is a regional highest if the value of f (x) at every point about x_0 does not exceed f (x_0). Similarly, a task f (x) is said to achieve its lowest value now, x = x_0 if:

$$f(x_0 + h) - f(x_0) > 0 \text{ or } f(x_0 + h) > f(x_0)$$

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When a task has numerous regional highest and lowest ideals, the international least (in case of cost minimization) or global highest (in case of profit maximization) is achieved by evaluating the value of the work at several intense times (including the limits of the domain). The global lowest value of a function is the most negligible value among all regional most inadequate standards of the task in the field. Similarly, the global highest value of a study is the highest among all regional highest values of the function in the domain. In Fig. 1, point E, i.e., f (x_4), represents the globalhighest, whereas point F, i.e., f ($x_{5,}$) represents the international least.

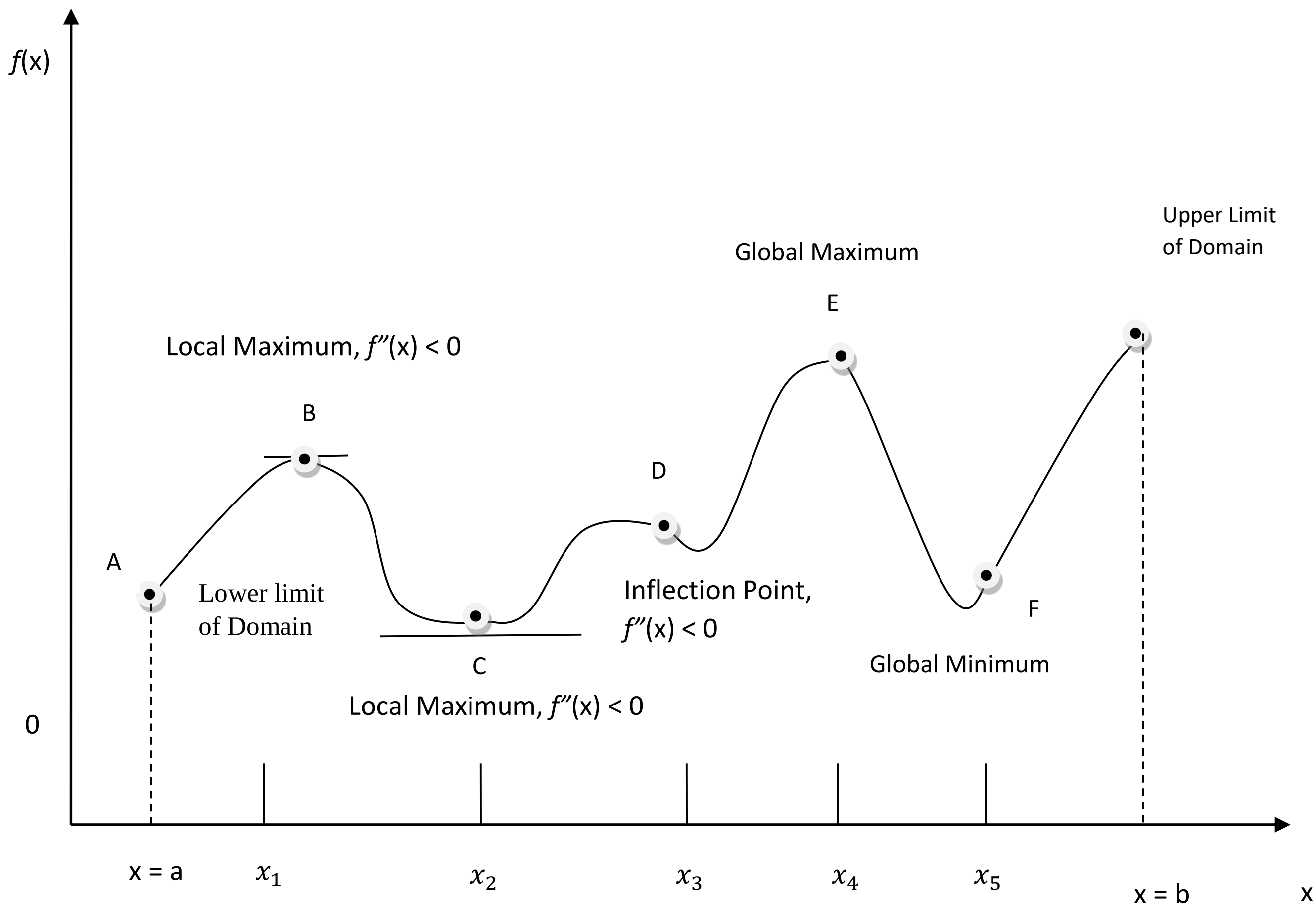

Figure-1 Regional and Global Optimum

The global highest (or lowest) over the larger interval can also occur at an endpoint of the period rather than at any regional (relative) highest or least point. A regional highest rate of a function can be less than a regional lowest value of the task.

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2.2 Circumstances for Regional Least and Highest Value

Proposition-1 - (Required condition) a critical situation for a pointx_0 to be the regional extreme (regional highest and lowest) of a utility y = f (x) defined in the interval $a \leq x \leq b$is that the first derivative of f (x) occurs as a limited figure at x = x_0 and f '(x_0)= 0.

Proof: Let y = f (x) be a given function that can be expanded in the neighborhoods of x = x_0 by Taylor's theorem. Let at x = x_0 the value of f (x) be f (x_0).

Consider two values of x, namely + h and – h, in the neighborhood and either side of x = x_0 (h being very small). If highest is at x =x_0, then from definition, f (x_0) >f (x_0 + h) and f (x_0) >f (x_0 – h).

⇒(x_0 + h) – f (x_0) and f (x_0 – h) – f (x_0) are both negative for highest atx =x_0 . Further, if least is at x = x_0,

⇒f (x_0) <f (x_0 + h) and f (x_0) <f (x_0 – h).

⇒f (x_0+ h) – f (x_0) and f (x_0 – h) – f (x_0) are both positive for least at x =x_0 .

By using Taylor's theorem, we have:

$$f(x_0+h) = f(x_0) + hf'(x_0) + \frac{h^2}{2!}f''(x_0) + \cdots + \frac{h^n}{n!}f^n(x_0) + R_n(x_0+\theta h);$$
$$0 < \theta < 1$$

$f(x_0+h) - f(x_0) = \ hf'(x_0) + \frac{h^2}{2!}f''(x_0) + \cdots$ (1)

Where $R_n(x_0+\theta h) = \frac{h^{n+1}}{(n+1)!}f^{n+1}(x_0+\theta h)$ is called the remainder.

The expressions $f'(x_0)$ and $f''(x_0)$ represent the first and second derivative of f (x) at x = x_0

$f(x_0-h) = f(x_0) - \ hf'(x_0) + \frac{h^2}{2!}f''(x_0) - \cdots$ (2)

$$f(x_0-h) - f(x_0) = -\,hf'(x_0) + \frac{h^2}{2!}f''(x_0) - \cdots$$

If h is very small, then neglecting the terms of higher order, we get,

$f(x_0-h) - f(x_0) = hf'(x_0)$ (3)

$f(x_0-h) - f(x_0) = -\,hf'(x_0)$ (4)

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For x = x_0 to be a regional highest or least value, the sign of f (x_0+ h) – f (x_0) and f (x_0 – h) – f (x_0) must be the same for all x = x_0± h.

Thus, from Eqns (3) and (4) if(x_0+ h) – f (x_0) and f (x_0 – h) – f (x_0) have the same sign, then$f'(x_0)$ should be zero; otherwise, they will have different characters.

Hence the required situation for any purpose f(x) to have regional optimum value at any extreme point x =x_0, is that its initial derivative f '(x_0) = 0.

Remark: The distinction between a regional lowest and regional highest can also be seen by examining the direction of change of the first derivative, f ′(x_0) at x = x_0

- If the sign-off ′(x_0) changes from positive to negative as x increases about x =x_0, then the rate of f (x) will be a regional highest =x_0, then the value of f (x) will be a regional highest.
- If the sign-off ′(x_0) changes from negative to positive as x increases about x = x_0, then the value of f (x) will be a regional least.

Proposition-2 :(Satisfactory condition) If at an extreme point x =$x_0$off(x), the first (n – 1) derivatives of it become zero, i.e.,f ′(x_0) = f ″(x_0) = . . . =$f^{n-1}(x_0) = 0$ and $f^n(x_0) \neq 0$ then:

(i) Regional highest off (x) occurs at x = x_0, if$f^n(x_0) < 0$, for n even,

(ii) Regional least off(x) occurs at x =x_0 , if $f^n(x_0) > 0$, for n even,

(iii) Point of inflection occurs at x =x_0, if $f^n(x_0) \neq 0$, for anodd.

Proof: From Theorem 1 at an intense point x = x_0, f ′(x_0) = 0. Then from Eqns (1) and (2),

We have

$$f(x_0 + h) - f(x_0) = \frac{h^2}{2!} f''(x_0) \quad (5)$$

$$f(x_0 - h) - f(x_0) = \frac{h^2}{2!} f''(x_0) \quad (6)$$

Neglecting powers of h higher than second. Here, the following three possible issues may occur:

Case 1: If f '' (x_0) > 0, then both f (x_0 + h) – f (x_0) and f (x_0 – h) – f (x_0) are positive and hence regional least value of f (x) exists at x =x_0.

Case 2: If f '' (x_0) < 0, then both f (x_0 + h) – f (x_0) and f (x_0 – h) – f (x_0) are negative and hence regional highest rate of f (x) exists at x =x_0 .

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Case 3: If f '' (x_0) = 0, then no information is obtained about the highest or least rate of f (x). In this case, the function f (x) may have a regional highest, a regional least, or a point of intonation. Hence, iff '' (x_0) = 0, then we examine successively higher-order derivatives off(x) at x = x_0 until we find a derivative such that$f^n(x_0) \neq 0,$ n ≥ 2.If$f^n(x_0) < 0,$ for n even, then f (x) has regional highest value at x =x_0 . If $f^n(x_0) < 0,$ for n even, then f(x) has regional least value at x =x_0 . If n is odd, then x = x_0 is the point of inflection (or saddle point).

The required and satisfactory circumstances for the presence of regional highest and least and point of inflection are summarized in Table1. The entire preceding discussion is summarized in Fig. 1.

Table 1:

Required Condition	Satisfactory Condition	Nature of Function	Conclusion
f '(x_0) = 0	f '(x_0) = f ''(x_0) = . . . =$f^{n-1}(x_0) = 0$ and $f^n(x_0) < 0,$ n even.	Concave	Regional highest at x =x_0
f '(x_0) = 0	f '(x_0) = f ''(x_0) = . . . =$f^{n-1}(x_0) = 0$ and $f^n(x_0) > 0,$ n even.	Convex	Regional least atx =x_0
f '(x_0) = 0	f '(x_0) = f ''(x_0) = . . . =$f^{n-1}(x_0) = 0$ and $f^n(x_0) \neq 0,$ n odd.		Point of inflection at x =x_0

It becomes easy to find the highest or least values when the function is either convex or concave. If a convex function, the first derivative set equal to zero must give at least one regional least. The value of the process at the endpoints of the domain may still be the global least. Similarly, if a function is concave, the first derivative set equal to zero must give at least one regional highest. It is due to this reason that functions most found in business are assumed to be either concave or convex.

Remarks

- A regional least of a convex purpose on a convex set is also a universal least of that function.
- A convex set's regional highest of a concave purpose is also a universal highest of that function.
- A regional least of a strictly convex purpose on a convex set is also a distinctive universal least of that function.
- A regional highest of a strictly concave purpose on a convex set is also a distinctive universal highest of that function.

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2.3 Optimizing Multi-variable Functions

To optimize a multi-variable purpose, we use the theory of restricted derivatives. This is because partial results measure the modification in the determined variable due to unit changing in one of the individual variables while maintaining the continuous influence of all additional individual variables. The required and satisfactory circumstances for regional optimum (highest or least) of unrestrained multi-variable functions may be described as follows:

Taylor's series expansion of a multivariable function:

Let f (x) be a highly treasured constant and differentiable purpose of x in E^n

Let (x + h) be a point about x such that

$h = \left(h_1, h_{2,} h_3 \; \dots \dots h_n\right)^T$ And $x = \left(x_1, x_{2,} x_3 \; \dots \dots x_n\right)^T$

$(x + h) = (x_1 + h_1, x_2 + h_2, x_3 + h_3 \dots \dots \dots x_n + h_n)^T \backslash$

Then f (x) can be conveyed as a power series linking f (x) 's differentials to Taylor's series.

$$f(x + h) = f\left(x_1 + h_1, x_2 + h_2, x_3 + h_{3,} \dots \dots . x_n + h_n\right)$$

$= f(x) + \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right) h_i + \frac{1}{2!} \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\frac{\partial^2 f}{\partial x_i \partial x_i}\right) h_i h_j +$
$terms \;\; involving \; higher \; powers \; of \; h$(7)

Now we describe the descent vector off(x), denoted by ∇f (x), as follows. The nth descent vector whose elements are the partial derivatives off(x) and Hessian matrix H(x) of order n calculated at x + q h (0 <q < 1)

Are as follow:

$$\nabla f(x) = \left[\frac{\partial f(x)}{\partial x_1}, \frac{\partial f(x)}{\partial x_2}, \dots \dots . \frac{\partial f(x)}{\partial x_n}\right]^T$$

And

$$H(x) = \begin{bmatrix} \frac{\partial^2 f(x)}{\partial x_1 \partial x_1} & \frac{\partial^2 f(x)}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f(x)}{\partial x_2 \partial x_1} & \frac{\partial^2 f(x)}{\partial x_2 \partial x_2} \end{bmatrix}$$

Using the above description, we can write:

$f(x + h) - f(x) = \nabla f(x) h + \frac{1}{2!} h^T H(x) h; x = x + \theta h \; and \; o < \theta < 1$ (8)

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3. CONSTRAINED MULTIVARIABLE OPTIMIZATION WITH EQUALITY CONSTRAINTS

This segment will discuss the difficulty of optimizing continuous and differentiable function subjectto equality constraints.

Optimize (max or min) $Z = f(x_{1,} x_{2,} x_{3,} \dots \dots \dots x_{n,})$

Subject to the constraints

$$h_i(x_{1,} x_{2,} x_{3,} \dots \dots \dots x_{n,}) = b_i \; ; i = 1,2, \dots \dots m$$

(Max or min) Z = f (**x**)

Subject to the restrictions

$$g_i(x) = 0, \quad i = 1,2, \dots \dots m \qquad (9)$$

Where

$$x = (x_{1,} x_{2,} x_{3,} \dots \dots \dots x_{n,}) \quad (10)$$

And

$$g_i(x) = h_i(x) - b_i \; ; \; b_i \; is \; a \; constant$$

Here it is assumed that m <n to get the solution.

(i) Directly Substitution Technique, and

(ii) Lagrange Multipliers Technique.

3.1 Direct Substitution Method

Since the restriction set $g_i(x)$ is constant and differentiable, any variable in the restraint set can be stated in conditions of the remaining variables. Then it is replaced with the objective function. The new purpose function, so acquired, is not subject able to any constraints, and therefore its optimum rate can be developed by the straightforward optimization process.

Occasionally, this process is inappropriate, especially when more than two variables are in the objective purpose and subject to constraints.

3.2 Lagrange Multipliers Method

In this process, an extra variable in each of the constraints is included. Therefore, if the question has n variable quantity and m equal opportunity restrictions, then the m variable different amount is to be improved to have n + m variables. Before discussing the general

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method, let us illustrate its salient characteristics through the following easy question that requires just three variables:

Required condition for a question with n = 3 and m = 1Consider the NLP problem

Optimize (max or min) Z= $f\left(x_{1,}x_{2,}x_3\right)$ (11)

Subject to the constraint

$g\left(x_{1,}x_{2,}x_3\right) = 0$(12)

Let an optimum value of Z occur at a point ($x_{1,}x_{2,}x_{3,}$) = (a, b, c) at which at least one of the incomplete derivatives$\frac{\partial g}{\partial x_1}\frac{\partial g}{\partial x_2},\frac{\partial g}{\partial x_3}$Thus, we may proceed as follows:

(i) Choose one variable say $x_{3,}$ in constraint (12) and express it in conditions of the remaining two variables such that $x_3 = h(x_1, x_2)$

(ii) Substitute the amount ofx_3 into the goal function (11). We then get:

Z = f {(x_1, x_2), h (x_1, x_2)}

From unconstrained optimization methods, we understand that the critical situation for regional optimum is that all first derivatives with respect tox_1 and x_2 must be zero.

$$\frac{\partial Z}{\partial x_j} = 0; j = 1,2 \qquad (13)$$

Applying the chain rule for differentiation on (13) we get :

$$\frac{\partial Z}{\partial x_j} = \frac{\partial f}{\partial x_j} + \frac{\partial f}{\partial x_3}.\frac{\partial h}{\partial x_j}; \quad j = 1,2$$

But from Eq. (12), we have

$$\frac{\partial g}{\partial x_j} + \frac{\partial g}{\partial x_3}.\frac{\partial h}{\partial x_j} = 0 \;\; j = 1,2$$

$$\frac{\partial h}{\partial x_j} = -\frac{\frac{\partial g}{\partial x_j}}{\frac{\partial g}{\partial x_3}}; \frac{\partial g}{\partial x_3} \neq 0, j = 1,2$$

At a point ($x_{1,}x_{2,}x_{3,}$) = (a, b, c) we have

Since optimum occurs at a point (a, b, c), we have

$$\frac{\partial Z}{\partial x_j} = \frac{\partial f}{\partial x_j} - \left[\frac{\partial f}{\partial x_3}.\left\{\frac{\frac{\partial g}{\partial x_j}}{\frac{\partial g}{\partial x_3}}\right\}\right] = 0 \; at(x_{1,}x_{2,}x_{3,}) \;=\; (a, b, c) \qquad (14)$$

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As$\frac{\partial g}{\partial x_3} \neq 0$, we define an amountλ, called Lagrange multiplier as given below. The value of λ represents the amount of change in the goal task due to the per-unit shift in the constraint limit,

$$\frac{\partial f}{\partial x_3} - \lambda \frac{\partial g}{\partial x_3} = 0, at\ (x_{1,} x_{2,} x_{3,}) = (a, b, c)$$

$$\lambda = \frac{\frac{\partial f}{\partial x_3}}{\frac{\partial g}{\partial x_3}}$$

Equation (14) $\frac{\partial Z}{\partial x_j} = \left(\frac{\partial f}{\partial x_j} - \lambda \frac{\partial g}{\partial x_j}\right) = 0, j = 1,2$(15)

$at\ (x_{1,} x_{2,} x_{3,})$ = (a, b, c) and the constrain equation

$g(x_{1,} x_{2,} x_{3,} = 0$ (16)

Is also satisfied at the extreme (or critical) points,$x_{1,}$ = a, $x_{2,}$ = b and $x_{3,}$ = c. The circumstances (14) and (15) are called required circumstances for a regional optimum, provided not all$\frac{\partial g}{\partial x_j}$ become zero at the extreme points.

$$L(x_j, \lambda) = f(x_j) - \lambda g(x_j),\ \ j = 1,2,3 \qquad (17)$$

We must, then, partially differentiate$L(x_j, \lambda)$ with respect to x_j(j = 1, 2, 3) and $\lambda$and equate them with zero. The following equations provide the required circumstances for regional optimum:

$$\frac{\partial Z}{\partial x_j} = \frac{\partial f}{\partial x_j} - \lambda \frac{\partial g}{\partial x_j} = 0, j = 1,2,3 \qquad (18)$$

$$\frac{\partial L}{\partial \lambda} = g(x_j) = 0, J = 1,2,3$$

These equations can be solved for the unknown x_j(j= 1, 2, 3) and λ.

Remark:

The necessary circumstances, so found, are adequate conditions for a highest (or least) if f (x) is concave (or convex), with equality constraints.

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4. CONSTRAINED MULTI-VARIABLE OPTIMIZATION AND INEQUITY RESTRICTIONS

The required and satisfactory circumstances for a regional optimal of the common nonlinear scheduling problem, with both equality and inequality restrictions, will be derived. The Kuhn-Tucker events (required as well as sufficient) will be used to derive optimality circumstances. Consider the following general nonlinear LP problem:

4.1 Kuhn-Tucker Required Conditions

Optimize Z = f (**x**)

Subject to the restraints

$$g_i(x) \leq 0, \text{ for i= 1, 2, . . ., m}$$

Where $x = (x_1, x_2 x_3 \ldots \ldots x_n)^T$and $g_i(x) = h_i(x) - b_i$

Add non-negative slack variablesg_i (i= 1, 2, . . ., m) in each of the restraints to convert them to equality constraints. The problem can then be restated as:

Optimize Z = g (**x**)

Subject to the restraints

$$g_i(x) + s_i^2 = 0 \text{ i = 1, 2, . . ., m}$$

The s_i^2 has only been added to ensure si's non-negative value (feasibility requirement)s_i and avoid adding s_i≥ 0 as an additional side constraint.

The new problem is the constrained multi-variable optimization question with equality restrictions with n + m variables. Thus, it can be resolved by using the Lagrangian multiplier method. For this, let us form the Lagrangian function as:

$$L(x, s, \lambda) = f(x) - \sum_{i=1}^{m} \lambda_i [g_i(x) + s_i^2]$$

Where $\lambda = \left(\lambda_1, \lambda_{2,} \lambda_3 \ldots \ldots \lambda_m\right)^T$

The required circumstances for an extreme situation to be regional optimum (max or min) can be obtained by deciphering the following equations:

$$\frac{\partial L}{\partial x_j} = \frac{\partial f(x)}{\partial x_j} - \sum_{i=1}^{m} \lambda_i \frac{\partial g_i(x)}{\partial x_j} = 0\,, j = 1,2, \ldots \ldots, n$$

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$$\frac{\partial L}{\partial \lambda_j} = -[g_i(x) + s_i^2] = 0, i = 1,2, \dots \dots . . m$$

$$\frac{\partial L}{\partial s_i} = 2x_i\lambda_i = 0 \ , i = 1,2, \dots \dots \dots . . m$$

The equation $\frac{\partial L}{\partial \lambda_i} = 0$= gives us back the original set of constraints: $g_i(x) + s_i^2 = 0$. If a restriction is satisfied with the equivalence symbol, $g_i(x)$=0 at the optimum point **x**, it is known as an active (binding or light) restraint, else it is known as an inactive (slack) constraint.

The equation $\frac{\partial L}{\partial s_i} = 0$, provides us the set of rules: $-2s_i\lambda_i = 0 \ or \ s_i\lambda_i = 0$for finding the unconstrained optimum. The condition $s_i\lambda_i = 0$implies that either$\lambda_i = 0$ or $s_i = 0$. If$s_i = 0$ andλ_i>0, then equation$\frac{\partial L}{\partial \lambda_j} = 0$ gives$g_i(x) = 0$. This means either $\lambda_i = 0 \ or g_i(x) = 0$, and therefore we mayalso write $\lambda_i s_i(x) = 0$.

Sinces_i^2 has been taken to be a non-negative (≥ 0) slack variable, therefore$g_i(x)$≥ 0. Hence, the equation $\lambda_i g_i(x) = 0$implies that when$g_i(x)$< 0,$\lambda_i = 0$ and when$g_i(x) = 0$,λ_i> 0. However,$\lambda_i$is unrestricted in sign corresponding to$g_i(x) = 0$

An optimum value of Z* because$1 = \frac{\partial Z}{\partial b_i} = 0$) and hence can be discarded.

Thus, the Kuhn-Tucker required circumstances (when active constraints are known) to be satisfied at a regional optimum (max or min) point to be indicated as follows.

Thus, the Kuhn-Tucker required circumstances (when active constraints are known) to be satisfied at a regional optimum (max or min) point be able to be indicated as follows:

$$\frac{\partial f}{\partial x_j} - \sum_{i=1}^{m} \lambda_i \frac{\partial g_i}{\partial x_j} = 0, \qquad j = 1,2, \dots \dots . . , n$$

$$\lambda_i g_i(x) = 0$$

$g_i(x) \leq 0$

$$\lambda_i \geq 0, \qquad i = 1,2, \dots \dots . , m$$

Remark: If the given problem is of minimization or if the constraints are of the form$g_i(x) \geq 0$, then $\lambda_i \leq 0$if the problem is of maximization with restraints of the structure$g_i(x) \leq 0$, then $\lambda_i \geq 0$

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4.2 Kuhn-Tucker Satisfactory Conditions

The Kuhn-Tucker required circumstances for the problem

Maximize Z = f (x)

Subject to the restrictions

$$g_i(x) \leq 0$$

Satisfactory iff (x) is concave and all $g_i(x) \leq 0$ are curved actions of x.

Proof: Maximize Z = f (x)

The restrictions

$g_i(x) \leq 0$,i= 1, 2, . . ., m

$$L(x, s, \lambda) = f(x) - \sum_{i=1}^{m} \lambda_i [g_i(x) + s_i^2]$$

If $\lambda_i \geq 0$, then $\lambda_i g_i(x)$ is convex and $-\lambda_i g_i(x)$ is concave. Further, since $\lambda_i s_i = 0$, we get $g_i(x) + s_i^2 = 0$. Thus, it follows that $L(x, s, \lambda)$ is a concave function. We have derived that a required term for f (x) to be a comparative highest at an extreme point is $L(x, s, \lambda)$ also have the same extreme point. Though, if $L(x, s, \lambda)$ is concave, its first derived must be zero only at one point, and obviously, this point must be an absolute highest for f (x).

5. CONCLUSION

The traditional optimization approaches achieve an optimal result of specific difficulties that involve continuous and differentiable functions. These procedures are logical, along with getting into the advantage of differential calculus to find maxima and minima points for a constrained single and multiple variable continuous functions. They denied multi-variable roles with equivalence and inequity restrictions. In this paper, circumstances for the regional and global least and highest value of an unconstrained objective function have been derived. The direct substitution method, Lagrange multipliers method, and Kuhn-Tucker method have also been discussed to obtain the optimal quantity of an empirical act with equivalence and variation restrictions.

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